Initial Equilibrium State and Parameters for
SFEcon's Model 1 Prototype

The prototype for SFEcon Model 1 operates on a hypothetical initial state for input/output structure of ten rows and columns. Sectors 1, 2, and 3 are generic industrial sectors that earn profits for distribution among household sectors. Sectors 4 and 5 are households (one rich and one not) that earn wages and have passive incomes from investments. Sector 6 is a third household sector that earns no wages and subsists on government transfers. Sectors 7 and 8 are governments (one national and one local) that process assets paid for by tax revenues. Sector 9 is a financial intermediary that manages the indebtedness of the poorer household sector 4 to the richer household sector 5. Sector 10 is another financial intermediary that manages the financial positions of the industrial sectors 1, 2, and 3 and directs dividends to both household sectors in proportion to their investments.

The economy's physical flows are organized in a (non-standard) I/O matrix: rows I correspond to sectors, columns J correspond to the goods that they use.

		Physical Flows E >			VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT	DPNDNT
					0	1	2	3	4	5	6
					0	I1	I2	I3	L1	L2	L3
			0	0	754374	-100.0	-750.0	-3200.0	-876.0	-8766.0	0.0
INDUSTRIES	CAPITAL	6.00%	1	I1	-71184	30.0	200.0	800.0	20.0	600.0	0.0
	GENERAL	6.00%	2	I2	-62183	25.0	100.0	700.0	30.0	600.0	0.0
	NON_DUR	6.00%	3	I3	-53889	20.0	150.0	500.0	20.0	600.0	0.0
HOUSEHOLDS	BOURGS		4	L1	-474642	22.9	228.6	996.1	781.1	198.5	0.0
	PRLTRT		5	L2	-77563	1.3	59.6	159.6	5.2	6569.4	0.0
	DPNDNT		6	L3	-1641	0.1	6.0	16.0	1.0	20.0	0.0
GOVERNANCE	LOCAL		7	G1	-4646	0.2	0.8	5.8	6.9	65.6	0.0
	NATIONAL		8	G2	-7077	0.3	0.9	8.2	10.6	100.0	0.0
BANKING	RETAIL		9	F1	-756	0.1	3.8	12.7	0.1	6.3	0.0
	COMMERCIAL		10	F2	-793	0.0	0.4	1.6	1.2	6.3	0.0
					0	I1	I2	I3	L1	L2	L3
					0	1	2	3	4	5	6
					VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT	DPNDNT

Figure 1: Physical Flows

Row I=0 expresses physical rates of the output realized via application of the physical inputs in the row J indexed by the output's position at [0,J]. Column J=0 receives the value of all goods J being used in a given row. Output and value are signed in the negative to indicate things that 'come back to us' out of the expenditure of assets.

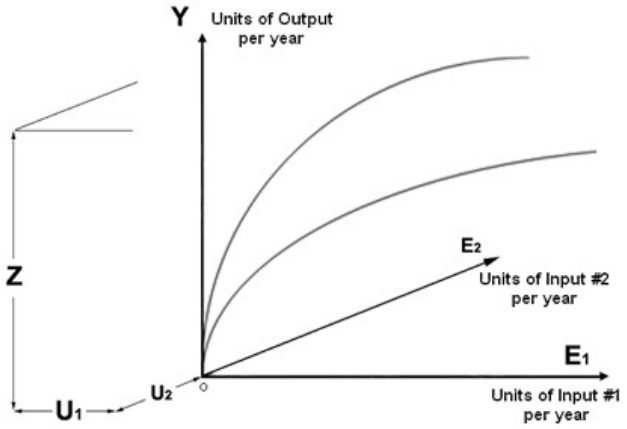
The relation between rates of physical flow and the respective sectors' rates of output are controlled by a matrix of parameters describing the sectors' technical and utility tradeoffs.

					INDUSTRIES			HOUSEHOLDS		
				VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT	DPNDNT
		Utility U >		0	1	2	3	4	5	6
				0	I1	I2	I3	L1	L2	L3
			0 0		1114.6	8335.0	35267.7	876.0	8766.0	100.0
INDUSTRIES	CAPITAL	1531144	1 I1		984.6	8511.0	42087.2	1412.3	75957.2	
	GENERAL	1333222	2 I2		858.5	7485.0	36643.4	1217.2	66061.1	
	NON_DUR	1144869	3 I3		738.6	6363.4	31567.7	1051.0	56643.4	
HOUSEHOLDS	BOURGS	63985	4 L1		19.5	135.4	796.2	-720.8	3000.8	0.0
	PRLTRT	15035	5 L2		8.7	25.9	261.5	8.8	-5815.9	0.0
	DPNDNT	281	6 L3		0.1	-4.4	-8.1	-0.7	-5.9	-47.7
GOVERNANCE	LOCAL	212	7 G1		0.0	0.4	0.1	-6.7	-54.9	
	NATIONAL	399	8 G2		0.0	1.4	3.0	-10.2	-80.1	
BANKING	RETAIL	110	9 F1		-0.1	-3.1	-9.7	0.0	-0.8	
	COMMERCIAL	45	10 F2		0.0	-0.1	-0.3	-1.2	-4.1	
				0	I1	I2	I3	L1	L2	L3
		ζ		0	1	2	3	4	5	6
				VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT	DPNDNT
					INDUSTRIES			HOUSEHOLDS		

Figure 2: Technical and Utility Parameters

As illustrated in Figure 3, these parameters specify the shape of an industrial sector's production tradeoffs in terms of a vector offset $[Z,U_J]$ between the production function's origin and point at which all the function's asymptotes intersect. ('Production functions' for the non-industrial sectors use these parameters in appropriately different interpretations.)

The interaction of each sector's utility parameters with commodity prices $[\pi,P_J]$ is expressed in its financial discriminant ζ_I \$/year. When the marginal value of all a sector's inputs equals the inputs' prices, all the terms in the equations on Figure 3's right compute to one and the same ζ_I.



$$\left[\pi Z \cdot \prod_{J}^{N} P_J U_J\right]^{\frac{1}{N+1}} = \pi \cdot (Z - Y)$$
$$\vdots$$
$$= P_J \cdot (U_J + E_J)$$
$$\vdots$$
$$= P_N \cdot (U_N + E_N)$$

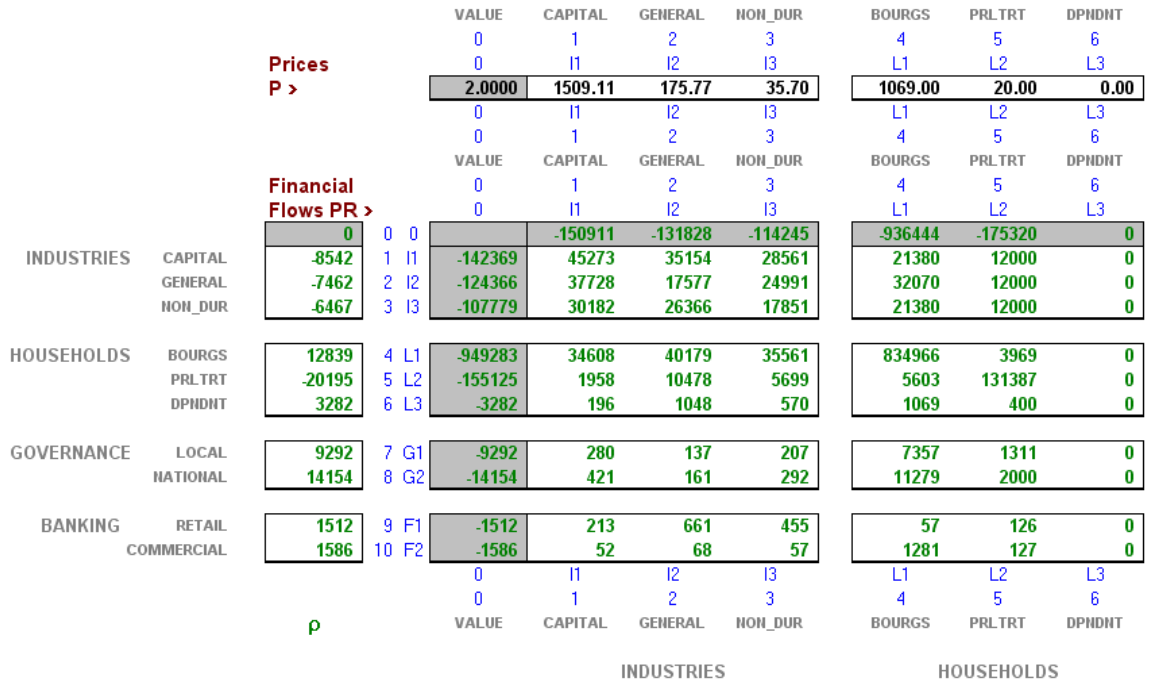
Figure 3: A Hyperbolic Production Function

Physical stasis in the SFEcon models arrives when physical rates of exchange interact with production functions to produce just enough of all outputs to replace that which is being used-up in creating those outputs. General optimality arrives when all the possible computations for every sector's ζ_I are identical.

This state is stable if the interaction of prices with the rates of asset replenishment determine constant levels of saving and investment for each sector. This interaction, shown in Figure 4, resolves to the financial profile ρ_I.

Negative values of ρ_I indicate that the industrial sectors 1, 2, and 3 are earning profits that are 'coming back to us' out of economic activity. The negative ρ_5 of $20,195/yr indicate that the proletarian household consumes less than it earns in wages plus passive income from investments in sectors 1, 2, and 3. These 'profits' are part of the bourgeois sector 4's passive income stream that are its reward for prior extensions of credit. Positive ρ_4 indicates that sector 4's consumption exceeds its income from wages by the amount of its passive income $12,829/yr.

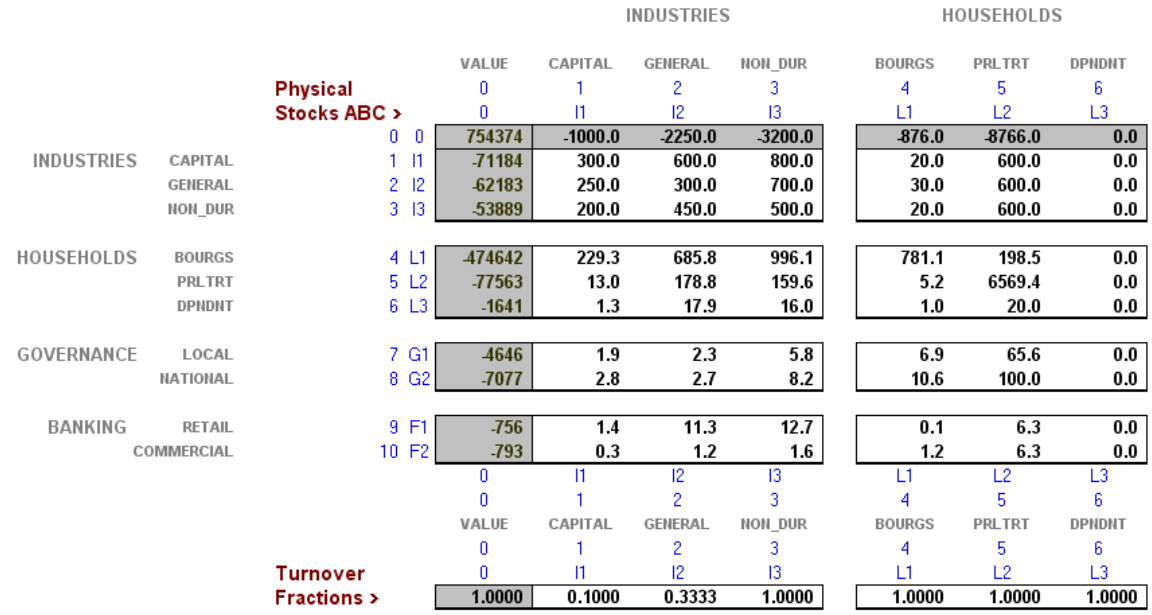
Note that each of the three industrial sectors' exhibits a return ($-\rho$) on asset turnover (in column 0) of 6.00%. Positive values of ρ_I for sectors 6 through 10 are 'passive incomes' originating in government transfers, taxes, and agents' fees for financial intermediation. These sectors do not create things that command a price or have calculable marginal costs of production. Model 1's prototype depicts a hypothetical state of isolation from foreign trade, hence the sum of all ρ_I's is always null.



				VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT	DPNDNT
				0	1	2	3	4	5	6
		Prices		0	I1	I2	I3	L1	L2	L3
		P >		2.0000	1509.11	175.77	35.70	1069.00	20.00	0.00
				0	I1	I2	I3	L1	L2	L3
				0	1	2	3	4	5	6
				VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT	DPNDNT
		Financial		0	1	2	3	4	5	6
		Flows PR >		0	I1	I2	I3	L1	L2	L3
		0	0 0		-150911	-131828	-114245	-936444	-175320	0
INDUSTRIES	CAPITAL	-8542	1 I1	-142369	45273	35154	28561	21380	12000	0
	GENERAL	-7462	2 I2	-124366	37728	17577	24991	32070	12000	0
	NON_DUR	-6467	3 I3	-107779	30182	26366	17851	21380	12000	0
HOUSEHOLDS	BOURGS	12839	4 L1	-949283	34608	40179	35561	834966	3969	0
	PRLTRT	-20195	5 L2	-155125	1958	10478	5699	5603	131387	0
	DPNDNT	3282	6 L3	-3282	196	1048	570	1069	400	0
GOVERNANCE	LOCAL	9292	7 G1	-9292	280	137	207	7357	1311	0
	NATIONAL	14154	8 G2	-14154	421	161	292	11279	2000	0
BANKING	RETAIL	1512	9 F1	-1512	213	661	455	57	126	0
	COMMERCIAL	1586	10 F2	-1586	52	68	57	1281	127	0
				0	I1	I2	I3	L1	L2	L3
				0	1	2	3	4	5	6
		ρ		VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT	DPNDNT
						INDUSTRIES			HOUSEHOLDS	

Figure 4: Financial Flows

Physical rates of input derive from their current stocking levels, which are the integrated difference between the rates at which they are replenished and the rates at which they are expended in production.



					INDUSTRIES			HOUSEHOLDS		
				VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT	DPNDNT
		Physical		0	1	2	3	4	5	6
		Stocks ABC >		0	I1	I2	I3	L1	L2	L3
			0 0	754374	-1000.0	-2250.0	-3200.0	-876.0	-8766.0	0.0
INDUSTRIES	CAPITAL		1 I1	-71184	300.0	600.0	800.0	20.0	600.0	0.0
	GENERAL		2 I2	-62183	250.0	300.0	700.0	30.0	600.0	0.0
	NON_DUR		3 I3	-53889	200.0	450.0	500.0	20.0	600.0	0.0
HOUSEHOLDS	BOURGS		4 L1	-474642	229.3	685.8	996.1	781.1	198.5	0.0
	PRLTRT		5 L2	-77563	13.0	178.8	159.6	5.2	6569.4	0.0
	DPNDNT		6 L3	-1641	1.3	17.9	16.0	1.0	20.0	0.0
GOVERNANCE	LOCAL		7 G1	-4646	1.9	2.3	5.8	6.9	65.6	0.0
	NATIONAL		8 G2	-7077	2.8	2.7	8.2	10.6	100.0	0.0
BANKING	RETAIL		9 F1	-756	1.4	11.3	12.7	0.1	6.3	0.0
	COMMERCIAL		10 F2	-793	0.3	1.2	1.6	1.2	6.3	0.0
				0	I1	I2	I3	L1	L2	L3
				0	1	2	3	4	5	6
				VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT	DPNDNT
		Turnover		0	1	2	3	4	5	6
		Fractions >		0	I1	I2	I3	L1	L2	L3
				1.0000	0.1000	0.3333	1.0000	1.0000	1.0000	1.0000

Figure 5: Physical Stocks

Expenditure rates for a given input J are governed by that input's turnover fraction V_J, multiplied by that input's level in the stock of a given sector I.

In SFEcon's analogy of the economic system as a huge, distributed analog computer, asset levels are the system's state variables: they are what the theory operates to change; and their continuity in time is what keeps the system on track and stable.

Turnover fractions also have application to the financial sectors; but the meanings they impart here differ radically from the obvious logistical sense of propelling physical asset stocks.

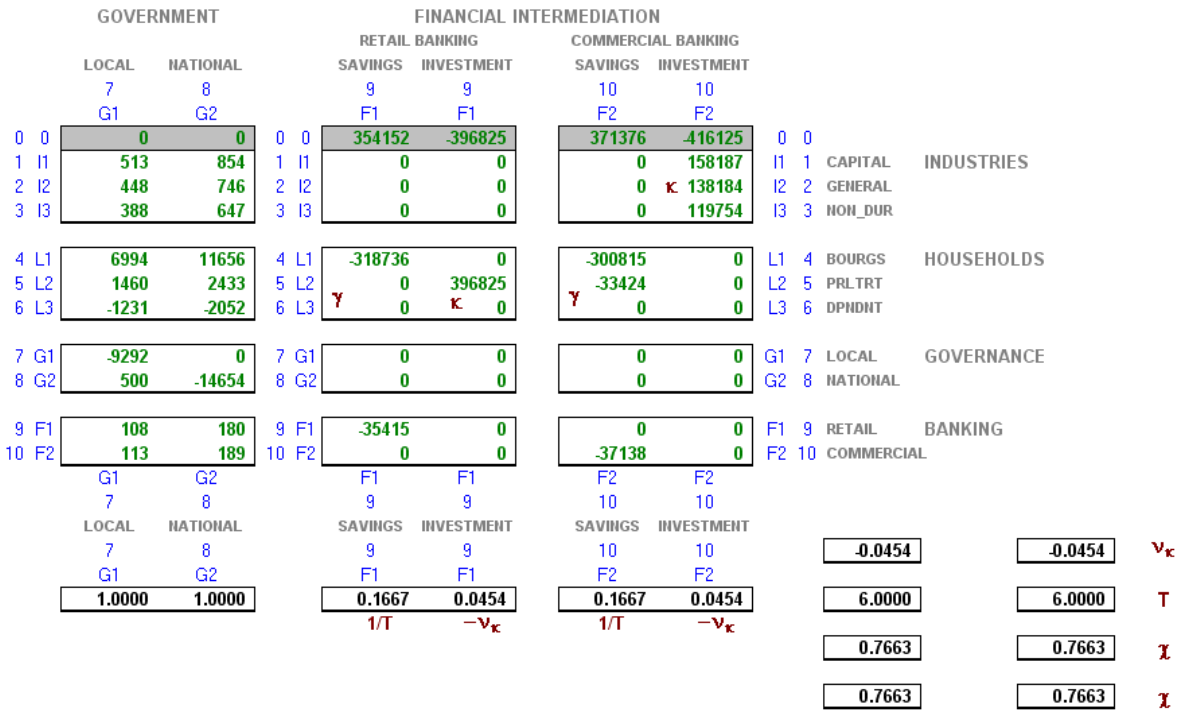


Figure 6: Monetary Stocks

SFEcon's input/output scheme uses governments' columns to record the taxes paid by the other sectors. Negative entries of -1231 and -2052 in the dependent sector 6's row effect transfer payments by which the dependent expenditures are offset. A negative sum of these payments is taken at a government's diagonal cell to actuate receipt of these taxes, thereby offsetting (to whatever degree) the government's expenditures as these are recorded in its row. The prototype's initial state has local governments tax incomes and corporate profits at 6%, and the national government taxing incomes and corporate profits at 10%. State and local governments initially have expenditures equaling their revenues with zero indebtedness.

As more vivid expressions of government finances enter later SFEcon models, there will of course arise a need for averages of past government interactions with the economy for the sectors to use in their financial planning. The government goods' 'turnover fractions' are parameters used in exponentially smoothing past rates of taxation; 'stocks' of the government good are the state variables upon which these averaging functions operate.

Each financial intermediary in an SFEcon model is allocated two columns: one for the dollar amount κ that the intermediary has invested among the sectors in which it specializes; and one for the dollar amount γ that the a household sector owns in terms of the savings instrument offered by the intermediary. Savings and investment need not be equal: $\kappa > \gamma$ indicates leverage, which is small but present in the prototype's initial state.

The entry 396825 in Figure 6 indicates that the proletarian sector 5 has been extended credit in that amount by the retail banking sector 9. Interest is owed on this amount. The entry -371876 indicates that the bourgeois sector 4 has entrusted savings in that amount to the retail banking sector. Interest is to be paid on this amount. The entry -35415 in the retail banking sector's row 9 is 10% of total saving 354152 in the retail banking sector, which is registered in row 0. This indicates that agents' fees are 10%.

Entries of 158187, 138184, and 119754 for the industrial sectors 1, 2, and 3 indicates that a total of 416125 dollars has been invested by the commercial banking sector 10. Interest is owed on this amount. Entries of -300815 and -33424 for the household sectors 4 and 5 indicate that the bourgeois and proletariat have entrusted savings in that amount to the commercial banking sector. Interest is to be paid on these amounts. The entry -37138 in the commercial banking sector's row 10 is 10% of total saving 371376 in the commercial banking sector, which is registered in row 0. This indicates that agents' fees are 10%.

This array of financial placements expresses a minimum of the competitive activities surrounding financial intermediation. The bourgeois has the option of investing in either the retail or commercial banking sectors. The commercial banking sector has the option of placing funds among three industrial sectors. Development of the prototype is aimed mainly understanding how intermediation resolves these competitions so as to keep the entire system stable.

Turnover factions for saving γ and investment κ are variable for each intermediary. In reference to savings, $1/T = 0.1667$/year indicates that savings are currently being committed for an investment term T of six years. In reference to investment, $-\nu = 0.0454$ means that interest for the use of capital is being charged at 4.54%/year.

Recalling that each industrial sector earns 6.00% per year on asset turnover, the initial interest rate of 4.54% is computed by deducting agents' fees of 10%, and tax rates of

10% and 6%, from the industries' 6.00% return. It is initially presumed that retail banking must also pay this rate in order to divert capital from its uses in industry to support of private consumption. (A more realistic model would of course feature returns on governments' financial instruments as the larger influence on interest rates; but government finances are kept minimal in Model 1 for the sake of realizing a fuller picture of financial intermediation.)

As in SFEcon Model 0, interest rates $-\nu$ and investment terms T are related through the idea of net present value χ:

$$\chi = \frac{1}{(1-\nu)^{T}}$$

Eqa. 1

This transcendental expression has an algebraic expression in terms of the financial state variables γ and κ (summed in row zero) of the SFEcon system:

$$\chi = \frac{\gamma/T}{\gamma/T - \nu \cdot \kappa}$$

Eqa. 2

With interest the rate $-\nu$ known, the investment term T is given by the elimination of χ from these equations. The prototype's initial state is contrived so that each intermediary's turnover fractions are equal.

Multiplying the financial sectors' current placements of savings and investment by their respective turnover fractions $-\nu$ and T produces the Monetary flows of Figure 7. These flows, being denominated in currency units, have a price of unity when they depict taxes and interest payments. Flows of savings are different in that their purchase of money is for delivery at the end of the investment term T. Purchase of a savings instrument is therefore at a price P_F that is discounted from unity per Equation 3:

$$P_F = \frac{1}{\chi} - 1$$

Eqa. 3

Figure 7 exhibits a P_F of 0.305 as the upshot of the intermediaries' initial turnover fractions. Multiplying Figure 7's monetary flows by their corresponding prices translates to the monetary transactions of Figure 8.

		GOVERNMENT		FINANCIAL INTERMEDIATION							
				RETAIL BANKING		COMMERCIAL BANKING					
		LOCAL	NATIONAL	SAVINGS	INVESTMENT	SAVINGS	INVESTMENT				
		7	8	9	9	10	10				
		G1	G2	F1	F2	F1	F2				
0	0	0	0	59025	-18000	61896	-18875	0	0		
1	I1	513	854	0	0	0	7175	I1	1	CAPITAL	INDUSTRIES
2	I2	448	746	0	0	0	6268	I2	2	GENERAL	
3	I3	388	647	0	0	0	5432	I3	3	NON_DUR	
4	L1	6994	11656	-53123	0	-50136	0	L1	4	BOURGS	HOUSEHOLDS
5	L2	1460	2433	0	18000	-5571	0	L2	5	PRLTRT	
6	L3	-1231	-2052	0	0	0	0	L3	6	DPNDNT	
7	G1	-9292	0	0	0	0	0	G1	7	LOCAL	GOVERNANCE
8	G2	500	-14654	0	0	0	0	G2	8	NATIONAL	
9	F1	108	180	-5903	0	0	0	F1	9	RETAIL	BANKING
10	F2	113	189	0	0	-6190	0	F2	10	COMMERCIAL	
		G1	G2	F1	F1	F2	F2				
		7	8	9	9	10	10				
		LOCAL	NATIONAL	SAVINGS	INVESTMENT	SAVINGS	INVESTMENT				
		7	8	9	9	10	10				
		G1	G2	F1	F1	F2	F2				
		1.0000	1.0000	0.3050	1.0000	0.3050	1.0000	P		0.3050	0.3050
								ι		0.0508	0.0508

Figure 7: Monetary Flows

With reference to row 0 of Figure 8, note that each intermediaries' totals of income from capital placements exactly offsets his payments to those holding his financial instrument, plus his agent's fee.

			GOVERNMENT		FINANCIAL INTERMEDIATION							
					RETAIL BANKING		COMMERCIAL BANKING					
			LOCAL	NATIONAL	SAVINGS	INVESTMENT	SAVINGS	INVESTMENT				
			7	8	9	9	10	10				
			G1	G2	F1	F1	F2	F2				
0	0	0	0	0	18000	-18000	18875	-18875	0	0		
-8542	0	I1	513	854	0	0	0	7175	I1	1	CAPITAL	INDUSTRIES
-7462	0	I2	448	746	0	0	0	6268	I2	2	GENERAL	
-6467	0	I3	388	647	0	0	0	5432	I3	3	NON_DUR	
12839	0	L1	6994	11656	-16200	0	-15289	0	L1	4	BOURGS	HOUSEHOLDS
-20195	0	L2	1460	2433	0	18000	-1699	0	L2	5	PRLTRT	
3282	0	L3	-1231	-2052	0	0	0	0	L3	6	DPNDNT	
9292	0	G1	-9292	0	0	0	0	0	G1	7	LOCAL	GOVERNANCE
14154	0	G2	500	-14654	0	0	0	0	G2	8	NATIONAL	
1512	0	F1	108	180	-1800	0	0	0	F1	9	RETAIL	BANKING
1586	0	F2	113	189	0	0	-1888	0	F2	10	COMMERCIAL	
			G1	G2	F1	F1	F2	F2				
			7	8	9	9	10	10				
ε			LOCAL	NATIONAL	SAVINGS	INVESTMENT	SAVINGS	INVESTMENT				
					RETAIL BANKING		COMMERCIAL BANKING					
			GOVERNMENT		FINANCIAL INTERMEDIATION							

Figure 8: Monetary Transactions

The prototype's initial state achieves its final aspect of general equilibrium in comparing the financial profile ρ generated by Figure 4's financial transactions with the financial profile ε generated by Figure 8's monetary transactions. Cell-for-cell equality in vectors ρ and ε indicate that prices are such that the profits and passive income demands arising from the exchange of physical assets are exactly absorbed by taxes, transfers, fees, and rewards/costs for financial placements.

Recall, per Figures 2 and 3, that these prices are also the values of marginal product for the sectors' inputs. Recall also the Figure 1's rates of asset usage are just sufficient (in their interactions with Figure 2's production parameters) to reproduce those asset usages. Taken together, these identities constitute general equilibrium in SFEcon modeling. When a model's parameters are held constant, a model should exhibit decaying orbits toward such a state.