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Economic Computation and SFEcon Model 0

– Kurt Roemer*
SFEcon

Abstract

Classical economics posits that material order owes to a general affinity between marginal revenues and marginal costs – which is of course just the sort of self-evident generality from which all Western science proceeds. The classical school does not, however, confer scientific status upon economics because its causality is thought to be so complicated as to forestall mechanistic expression in terms of a determinant mathematical system.

SFEcon Model 0 is offered as a direct counter-example to the indissolubility asserted for what economists commonly refer to as their 'economic computation' or 'socialist calculation' or 'Vienna' problem. This algorithm controls the continuum of all chaotic physical and financial states, as well as disequilibrium prices, by which an economic system might efficiently guide itself into a new, previously unknown, unique, and equifinal Pareto optimum.

The matrix structure of SFEcon models can accommodate any number of sectors and commodities organized into any number of national economies; and a models' transient responses to any combination of stimuli can be examined at any level of detail. The algorithm's boundary conditions are the shapes of sectors' production and utility tradeoffs; they express a degree of curvature for each cell of an international input/output array.

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Economic Computation and SFEcon Model 0

I. Obstacles

1. Economic Man

While every economist is likely to have professed that exhibitions of economic control sourced in general optimality are unrealizable,[1] a substantial body of our guild goes further in asserting that, even if such demonstrata were available, they would be irrelevant to the study of material affairs. Heterodox reaction to orthodox neoclassicism proceeds in all directions from a shared conviction that the marginalist premise itself is without application. Thus Economics as causal science (a matter of some distinction from political arithmetic) is currently split between its orthodox, neoclassical mainstream and its heterodox dissidents. Two contending premises encapsulate their positions:

- Neoclassicism attributes the orderliness of a macroeconomic system to its natural tendency toward general optimality. Optimizing behavior presupposes the activity of an 'economic man' devoted entirely to gain and endowed with perfect knowledge.

- Scientific analysis of human behavior has established that neoclassicism's economic man does not occur in sufficient proportions for his behaviors to register in studies of familiar, everyday interactions.[2] And actual businesses have long been observed operating at considerable distances from the condition of marginal revenues equaling marginal costs.[3,4]

While economic orthodoxy struggles to dissemble the second observation above, heterodoxy asserts that it is sufficient to falsify the first. No economic discourse can proceed except by re-enforcing one position while disparaging the other.

One resolution of this impasse challenges an axiom tacitly accepted by both contending parties, viz.: foundational microeconomics – a presumption that the behaviors observed for interpersonal and business relations must also govern the macroeconomic whole. If this presumption is valid, then orthodox macroeconomics is indeed falsified by behavioral psychology's inability to produce a specimen of neoclassicism's economic man.

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[1] Mises, Ludwig (von): "Die Wirtschaftsrechnung im sozialistischen Gemeinwesen" *Archiv für Sozialwissenschaften*, vol. 47 (1920).
http://mises.org/pdf/econcalc.pdf

[2] Kahneman, Daniel: "A Psychological Perspective on Economics." *American Economic Review* 93 (2), 2003

[3] Simon, Herbert A.: "Theories of Decision Making in Economics and Behavioral Science". *American Economic Review* June 1959, vol. xlix, #3

[4] Machlup, Fritz: "Theories of the Firm: Marginalist, Behavioral, and Managerial". *American Economic Review* March 1967, vol. LVII, #1

Foundational microeconomics might be called into question by observing its continuation of medieval thought: the Scholastic mind longed for God-like truth in which the *uni*verse might be viewed as *uni*fied; in which any truth, to be true, had to be operative in application to phenomena when viewed at any level of abstraction. Familiar scientific practice outgrew this preoccupation in the 14th Century.

The Scholastic premise that the whole must exactly equal the sum of its parts, being a mere definition, cannot be sensibly questioned. But such findings do nothing to detract from ordinary observations that **the behavior of the whole cannot be same as the behavior of any of its parts**. No actual system exists that contravenes this point. Automobiles are not composed of smaller parts that are individually capable of self-propelled navigation across terra firma. A laboratory rat is inarguably the sum of its constituent cells; but living cells' behaviors have nothing in common mammalian behavior with respect to locomotion, nourishment, reproduction, or any other life characteristic.

Science outside of economics has long contented itself with creating metaphors that are valid in certain degrees for certain applications. They accept that observations taken by instruments having different lengths of focus will reveal different phenomena requiring their specific and unique explanatory models, e.g.: Schrödinger's wave equation explains the periodic chart of elements, but it has no grasp of the Newtonian world that is familiar to us; and Newton's mechanics have no application in the sub-atomic world.

(While it is true that unified field theories are sought-for in the hard sciences, these efforts have yet to be productive, and might never constitute anything more than an acceptably modern way of seeking after the Deity.)

The absence of orthodoxy's economic man among the subjects of behavioral studies (usually college sophomores) might be reasonably attributed to economic man's comparative rarity among the population. It is not difficult to imagine these searches being more productive if conducted on Wall Street or in the City of London, where a minute labor specialty's placements of capital operates to create overall efficiency in the macroeconomic system.

Accepting that capitalism is a means for coordinating a great many labor specialties, with each vocation having a personality of its own, it should not be difficult to project the securities analyst into the role of homo economicus. He is preoccupied by gain; and, as often as not, personally hedonistic. His placements of capital among sectors are undertaken solely for the sake of maximal returns. And he has proprietary knowledge of the firms in his sector because he is inevitably among these firms' directors.

Thus neoclassical causality does not by any means require that everyone embody his own specimen of economic man – only that such creatures exist in the comparatively small numbers required at those places where general economic optimality can be created.

2. <u>The Knowledge 'Problem'</u>

Immediately upon accepting that the neoclassical premise is worthy of consideration as a source of economic order, one encounters neoclassicism's sanctification of its causality by placing it beyond the possibilities of human understanding:

> *It will be evident, even in the socialist society, that 1,000 hectolitres of wine are better than 800, and it is not difficult to decide whether it desires 1,000 hectolitres of wine rather than 500 of oil. There is no need for any system of calculation to establish this fact: the deciding element is the will of the economic subjects involved. But once this decision has been taken, the real task of rational economic direction only commences, i.e. economically, to place the means at the service of the end. That can only be done with some kind of economic calculation. The human mind cannot orientate itself properly among the bewildering mass of intermediate products and potentialities of production without such aid. It would simply stand perplexed before the problems of management and location*.[5]

Hayek's famous 1945 paper is most often cited as disparaging the possibility of economic calculation based on such conclusions:

> *The peculiar character of the problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. The economic problem of society is thus not merely a problem of how to allocate "given" resources – if "given" is taken to mean given to a single mind which deliberately solves the problem set by these "data." It is rather a problem of how to secure the best use of resources known to any of the members of society, for ends whose relative importance only these individuals know. Or, to put it briefly, it is a problem of the utilization of knowledge which is not given to anyone in its totality*.[6]

But a plain reading of this passage imputes nothing more than a statement of the problem that a science named 'economics' has nominated itself to solve.

In 1945 Hayek had nearly a half century of productive inquiry left to him; and we know that he spent these decades pondering how *the spontaneous ordering of markets* might be understood rationally. Such things are not done in anticipation of failure. Even Mises' latter-day acolytes concede Mises' dissatisfaction with supernatural explications of economic order . . .

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[5] Mises, Ibid.
[6] Hayek, Friedrich August (von): "The Use of Knowledge in Society". *American Economic Association: American Economic Review*. XXXV, No. 4, 1945, pp. 519-30

Ludwig von Mises didn't like references to the "miracle" of the marketplace or the "magic" of production or other terms that suggest that economic systems depend on some force that is beyond human comprehension. In his view, we are better off coming to a rational understanding of why markets are responsible for astounding levels of productivity that can support exponential increases in population and ever higher living standards.[7]

The Vienna problem is precisely one of understanding how partial, distributed knowledge can organize itself even though it cannot possibly be assembled at a single point, nor reacted-upon via instructions from any sort of central directorate. The notion of central direction is most especially ridiculous in its requirement that ever-changing economic factoids would have to arrive at their point of focus continuously, their implications calculated instantly, and then given expression with military punctilio.

As in the matter of foundational microeconomics, we again suggest that the path forward is concealed by tacit acceptance of an unexamined axiom, viz.: that order can only be attributed to information's having achieved focus prior to being acted-upon. And again we find no justification other than to continue scholasticism's attribution of the order in creation to the will of Hayek's omnipotent, inscrutable, *single mind*. Economic science, if there is to be such a thing, must grasp the economy itself **as** a single mind, rather than as the creature **of** any sort of exterior single mind.

The mundane science of more recent centuries has become quite adept in demonstrating how the dissociated motives of a system's constituent parts can indeed add-up to elegant behaviors in the system's whole – these having no visible counterpart in the petty motives by which they are created. The homeostasis of a healthy organism, for example, is understood as the confluence of cross-related biological subsystems processing local stimuli while having no awareness of the organism they compose.

In greater relation to our subject, we note that macroeconomic superorganisms have been with us since the Triassic Period. Beehives, termite mounds, and ant colonies achieve their homeostasis by continuously re-directing the efforts of millions of inhabitants toward their mutual overall material well-being. Obviously these vast 'social orders' do not proceed from diktat, which would be impossible for the insect mind to process. Order would not exist among merely instinctual beings except as the import of countless private pursuits in response to exceedingly simple incentives.

If the social Leviathans artificially created by primates might be taken as analogous the superorganisms of insects, then societal homeostasis might well be accounted-for by general optimality. Profit is a simple, persuasive incentive; and general optimality is defined as a state in which all opportunities for extraordinary profits have been resolved.

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[7] Rockwell, Llewellyn H. (Jr.), "The Faith of Entrepreneurs" *Mises Daily*, 23 December 2005 http://mises.org/daily/1990. Rockwell, however, concludes this piece by contradicting his mentor with *Mises forgive me: this is a miracle*.

3. Polynomial Factoring

Hayek's statement of the Vienna problem (there are numerous variations) provides our point of departure because it remains authoritative for the greater number of economists while achieving admirable quantitative precision:

> *The conditions which the solution of this optimum problem must satisfy have been fully worked out and can be stated best in mathematical form: put at their briefest, they are that the marginal rates of substitution between any two commodities or factors must be the same in all their different uses*.[8]

Invocation of *marginal rates of substitution* presupposes references to loci of technical indifference, which immediately brings us upon the categorical impediment most often cited against the possibility of formal economic computation, i.e.: the polynomial factoring problem.

> Descriptions of technical indifference giving rise to economic calculation must present diminishing marginal utility with respect to each of N inputs.
>
> A description of general optimality would designate a point on each of N economic actor's utility functions.
>
> Computation of the optimum must somehow solve N systems of N+1 equations in N+1 unknowns, were each equation would be of degree N+1.

General solutions to such systems have always been thought impossible given that there is no general method for factoring polynomials of degree greater than four; and general polynomials of even the second degree are likely to have imaginary numbers among their roots. If general optimality exists in nature, even in the immaterial sense of an attractant, there must exist at least one polynomial expressive of economic optimality that can always be factored, irrespective of the degrees of curvature it presents.

Though SFEcon's solution to the factoring problem has had limited acceptance, it is concise to a point where it can be presented for evaluation in the course of introducing SFEcon's main variables.

Hyperbolic shapes, because of their unique properties in the derivative, suggest themselves wherever a natural process of dynamic accumulation and development requires description. Figure 3-1 presents a hyperbolic relation between a sector's output rate Y and the input rates E_J of its productive factors J in the three dimensions that limit our visual imaginations. Loci of technical optima $[Y,E_J]$, are shaped by a set of utility parameters $[Z,U_J]$ so as to express diminishing marginal utility at every point. The set $[Z,U_J]$ relates a production or utility function's origin to the point at which all asymptotes to the hyperbolic form intersect.

[8] Hayek, Ibid.

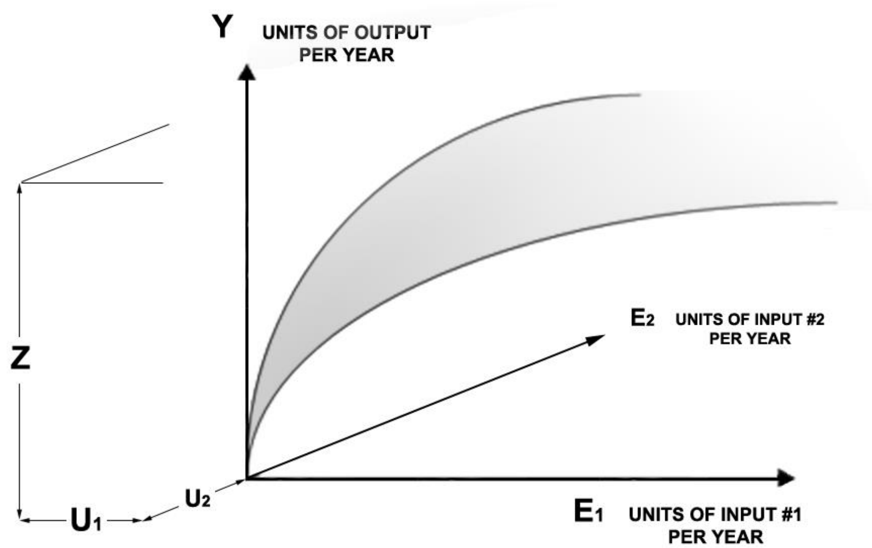

Figure 3-1: Hyperbolic Loci of Technical Indifference

For this parametric system, an economic sector's physical output rate Y would relate to its N physical input rates E_J by the following equation:

3-1) $$Y = Z \cdot \left[1 - \frac{U_1 \cdot U_2 \cdot \ldots \cdot U_N}{(U_1 + E_1) \cdot (U_2 + E_2) \cdot \ldots \cdot (U_N + E_N)}\right]$$

And the hyperbolic system's marginal products would then be disclosed thusly:

3-2) $$\frac{\partial Y}{\partial E_J} = \frac{Z - Y}{U_J + E_J}$$

Solution to the factoring problem proceeds from knowledge of the price environment $[\pi, P_J]$ where π is the price of the good being produced, and P_J represents a vector of prices for the factor inputs J. Optimal criteria require that each input's value of marginal product $\pi \cdot \partial Y / \partial E_J$ equal the price P_J of the input commodity. Stated in terms of the hyperbolic system's Equation 3-2 for marginal product, this means . . .

3-3) $$\pi \cdot \frac{Z - Y}{U_J + E_J} = P_J$$

. . . for all J = 1 to N. The optimal relationship between [Y,E_J] and [Z,U_J] would be governed by N such equations where N is the number of inputs. A simultaneous system relating [Y,E_J] with [Z,U_J] at [π,P_J] is completed by requiring that the optimal [Y,E_J] cooperate with [Z,U_J] in an exact solution to the production function of Equation 3-1.

Any such non-trivial production function that is properly expressive of diminishing marginal utility will present enough degrees of curvature so as to contra-indicate the possibility of polynomial factoring. The hyperbolic system does, however, offer a general, exact, closed-form, and almost certainly unique solution to the specific problem of economic optimality. It unfolds by identifying ζ, the central equality in Equation 3-3's relation of marginal value to price:

3-4)
$$\begin{aligned} \zeta &= \pi \cdot (Z - Y) \\ &= P_1 \cdot (U_1 + E_1) \\ &= P_2 \cdot (U_2 + E_2) \\ &\vdots \\ &= P_N \cdot (U_N + E_N) \end{aligned}$$

Solution proceeds by rearranging Equation 3-1's production function:

3-5)
$$1 = \frac{Z}{(Z - Y)} \cdot \frac{U_1}{(U_1 + E_1)} \cdot \frac{U_2}{(U_2 + E_2)} \cdot \ldots \cdot \frac{U_N}{(U_N + E_N)}$$

Unit ratios [π/π, P_J/P_J] are then applied to each corresponding term in Equation 3-5:

3-6)
$$1 = \frac{\pi \cdot Z}{\pi \cdot (Z - Y)} \cdot \frac{P_1 \cdot U_1}{P_1 \cdot (U_1 + E_1)} \cdot \frac{P_2 \cdot U_2}{P_2 \cdot (U_2 + E_2)} \cdot \ldots \cdot \frac{P_N \cdot U_N}{P_N \cdot (U_N + E_N)}$$

Equation 3-4 provides substitutions of ζ for each denominator in the above:

3-7)
$$1 = \frac{\pi \cdot Z}{\zeta} \cdot \frac{P_1 \cdot U_1}{\zeta} \cdot \frac{P_2 \cdot U_2}{\zeta} \cdot \ldots \cdot \frac{P_N \cdot U_N}{\zeta}$$

Solving for ζ then yields:

3-8)
$$\zeta = (\pi \cdot Z \cdot P_1 \cdot U_1 \cdot P_2 \cdot U_2 \cdot \ldots \cdot P_N \cdot U_N)^{\frac{1}{N+1}}$$

This calculation of ζ serves to isolate Equation 3-4's economic optimum from among the technical optima of Figure 3-1. If an economic sector's price environment [π,P_J] and utility tradeoffs [Z,U_J] are known, they determine the optimal values of [Y,E_J] in mathematically closed-form. So there does exist one polynomial expressive of economic optimality that can always be factored (by simple extraction of a higher-ordered root) irrespective of the degrees of curvature it expresses.[9]

4. Prospect

SFEcon's critique of neoclassicism asserts neither non-computability of the optimum nor deficient causality. We assert only a degree of misapplication: solution to the Vienna problem perforce requires that all economic actors be 'economic' in the sense of producing a recognizable product having a calculable marginal cost of production. Our Model 0 embodies this deficiency insofar as its economic actors must present themselves as either generic industrial sectors seeking to maximize profits, or as household sectors seeking to equate the utility of their last hour worked with their first hour at leisure. This leaves at least three critical matters unresolved:

> Robust financial intermediation does occur in Model 0, but the intermediary himself consumes no assets in a wholly disinterested process of optimizing capital placements.
>
> Only one intermediary is permitted to each national economy, which limits financial instruments to a single portfolio for each nation's totality of producing assets.
>
> Dependent populations, and the government sectors effecting the transfer payments on which they depend, are subsumed in the operations of households.

With intermediaries, governments, and dependent populations becoming increasing important economic actors, their demands on the economic system require distinct expressions that are certainly beyond marginalism's capabilities to portray. But the non-economic sectors' consumption of assets, with its consequent impact on prices, necessitates their differentiated presence in useful systems of economic calculation.[10]

Model 0's embodiment of pure capitalism is perhaps best understood as normative rather than descriptive. Reduction of neoclassical causation to a general and determinant mathematical system begins to be productive when the dynamics of elementary economic adjustment are modeled through familiar emulation technologies.

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[9] SFEcon has computed a great many hyperbolic expressions of economic potential from historical series of input/output data, and found them to be quite stable on the dimension of time. Details are available at www.sfecon.com.

[10] Such models exist within the SFEcon scheme, and can be examined at www.sfecon.com; but these are not pertinent to the Vienna problem being treated here.

These emulators would then constitute a theoretical chassis with which to uphold models that achieve useful realism by incorporating non-economic sectors.

It is to be hoped that acceptance of such models might reconnect the heterodox impulse toward hard science with the orthodox mainstream of economics. This would require that heterodox macroeconomists reconsider neoclassicism as a limited but solid foundation upon which to build, rather than a solid but erroneous impediment to building anything useful.

II. Apparatus

5. A Desktop Prototype

Claims that economic order can be realistically portrayed in terms of decaying orbits around a calculable general economic optimum will remain questionable until they are reduced to a determinant mathematical system. The possibility of such a system has been disparaged because it would be conceptually vast:

> It would necessarily abstract prices and markets into a spatial structure that can be segmented into any desired scheme of interacting economies, economic sectors, and commodities.
>
> And it would be dynamic in exhibiting the place that money and capital have in controlling efficient, stable adjustments to changes in manufacturing technique, resource availability, demographics, etc.

There is obviously no point in offering such a system unless it can be accompanied by a mechanism capable of animating its slate of equations to reveal their intrinsic behaviors.

Fortunately these mechanisms are already familiar in the form of kinetic videogames. Where the videogame validates the mechanics by which engineers understand material reality, so might an economic videogame validate the premises thought to command economic adjustment.

A companion Excel® workbook provides the numerical content referenced by this monograph. It can be downloaded from:

www.sfecon.com/M0.3.2.3.xlsm

This workbook emulates economic adjustment for a hypothetical global system. Three economies inhabit this world; and they are all described by the same input/output structure. Industry is segmented among three sectors producing, respectively, capital goods, general goods, and non-durable goods. And each economy is populated by two household sectors, viz.: a bourgeoisie and a proletariat. Each sector uses all the others' outputs; so each of the model's fifteen utility functions has five degrees of curvature – thereby inviting the mathematical crises thought to arise from the polynomial factoring problem. As Model 0's program is to represent the purest neoclassicism, no government sector is specified; and the actions of governance must be subsumed in the household sectors. Again: general SFEcon theory imposes no limit on the matrix segmentations that might be given to full-scale macroeconomic models.

It is essential to have this workbook available for reference on a convenient Windows® system as you study Model 0. Two considerations should be borne in mind when acquiring the workbook:

- This workbook was originally written in Excel 2003, and has been tested in all subsequent editions through Excel 2013. The workbook contains VBasic programs that will alert anti-virus software.

  If opened in Excel 2003, your 'SECURITY SETTING' must be no higher than 'MEDIUM' when the workbook is opened and macros must be 'ENABLED' in order to run the programs.

  If opened in later editions of Excel, 'EDITING' and 'CONTENT' must be 'ENABLED' if and when your security settings offer those options.

- The original numerical content of the workbook should be preserved in a separate file because these quantitative specifics are referenced throughout the exposition to follow. Should the original workbook be written over, a fresh one can be downloaded from sfecon.com.

The demonstration workbook contains nine worksheets. The first three sheets following its READ ME tab establish an initial I/O state for the experiments that follow:

- GPE allows entry of an initial set of physical input/output rates, and computes the corresponding general equilibrium prices.

- UTILITY computes the curving, hyperbolic production and utility tradeoffs for which the physical I/O pattern is optimal at the general equilibrium prices.

- ISTATE computes the initial state variables implied by whatever physical I/O matrix and prices are present on the GPE sheet.

As delivered, the workbook already contains a perfectly serviceable initial state; so there is no need for the auditor to examine these first three sheets in any detail until his acquaintance with SFEcon is well established.

All of the subsequent five sheets are portals into the numerical specifics of the emulation process.

- ECON 1, ECON 2, and ECON 3 each describe one of the three economies K in the hypothetical global system.

- GLOBAL organizes physical interchanges of the commodities J within the economies K.

- FINANCIAL presents the several monetary flows surrounding the sectors I in the currencies of the economies K, as well as the financial discriminants ζ_{IK} that constitute the inmost financial identities of each sector I in each economy K.

The experiments to be performed with our workbook are dynamic, presenting all the chaotic states and disequilibrium prices by which the global economic system maneuvers itself into a new, unique, and equifinal general optimum. These data are, however, presented in a tabular form most suitable for a ‘comparative statics’ style of analysis. The most instructive use of this workbook is therefore to establish all the criteria for the general optimum, and to demonstrate that these criteria will always come into focus if the model is allowed to operate on constant parameters for a sufficient period.

Ultimately, the experimenter should become convinced that these results ensue irrespective of the numerical content on which the model operates, the extent to which its input/output structure might be segmented, or the path the model has been on prior to the point at which its parameters ceased to change.

Because the criteria for general optimality are complicated, their presentation has required the rather extensive, 5x5x3 demonstration matrix of the companion workbook. Those experimenters wishing to go on to an understanding of the dynamics portrayed here will find more suitable exemplars of our technology at sfecon.com. The models available there operate on more aggregated matrices, present their outputs in graphic form, and reveal the emulation algorithm through manageable spreadsheets.

6. Simulation Controls

The demonstration’s controls can be accessed from any of the five worksheets constituting an experimental portal, viz.: ECON 1, ECON 2, ECON 3, GLOBAL, and FINANCIAL. As shown in Figure 6-1, these controls consist of three buttons centered at the top left of each worksheet, together with a slider down the middle.

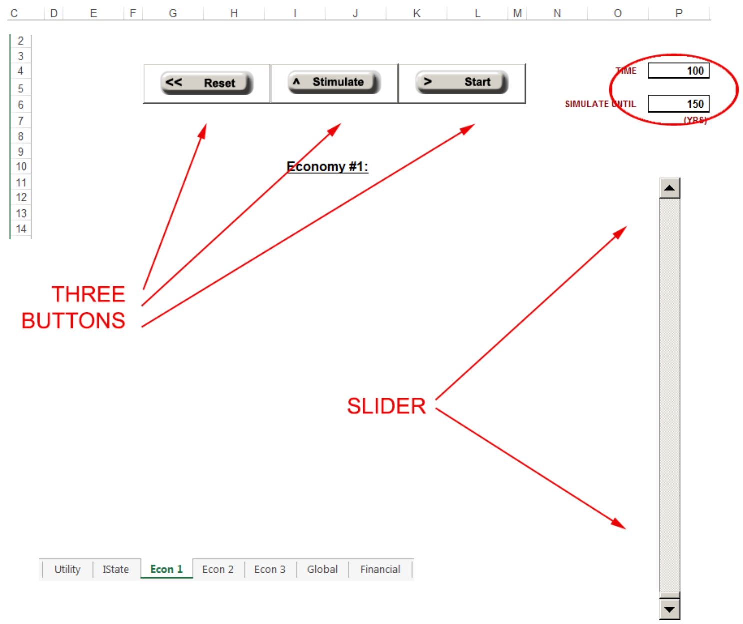


Figure 6-1:
Simulation Controls

- The RESET button creates an experiment's initial, general equilibrium state. In this state, the three hypothetical economies are identical, and therefore have no potential for interaction. **The model must be RESET at some point after the workbook is loaded into Excel and prior to running a simulation.**

- The STIMULATE button makes economy #2 a more efficient producer of good #2, and economy #3 a less efficient producer of good #3. This button can be pressed whenever the emulation has stopped, and from any of the five experimental portals. After STIMULATE has been pressed once in any portal, it has no further effect until the model is RESET.

- The START button launches an emulation from the system's state at the current value of TIME. A quantitative description of the system's state will be reported-out for each ensuing year until the SIMULATE UNTIL year has been reached. Current values for TIME and SIMULATE UNTIL are reported in the red oval of Figure 6-1.

- The slider at the diagram's middle can be run up and down to increase or decrease the interval between TIME and SIMULATE UNTIL. TIME is reset to zero whenever the model is RESET or STIMULATEd.

The controls' design is intended to allow repeated runs of the same experiment so that it might be analyzed from different experimental portals and with focus on different variables. Note that you can change experimental portals whenever the emulator stops, and then resume emulation from the new portal. The workbook itself is structured to allow an experimenter to create other, perhaps more complex patterns of stimulation for the model to resolve. Such experiments are encouraged: the means for their construction will become apparent in the course of explaining the theory behind SFEcon; and details as to how the model might be stimulated are in the appendices.

7. An Initial Experiment

It is suggested that our experimental apparatus be run through its standard demonstration before entering into a detailed discussion of the theory behind these quantitative results. Once again, be advised that the original numerical content of this Excel workbook should be preserved because these specifics are referenced throughout the exposition to follow.

To perform an initial shake-down run of the apparatus, go to the workbook's ECON 1 tab. The simulation controls described in Figure 6-1 above can be found toward the upper left corner of this worksheet, together with two matrices organized in input/output format:

- 'Physical I/O' contains the physical replenishment rates R_{IJK} at which the sectors I of economy K=1 are taking the commodities J off the market in order to replace the assets currently being used-up in creating the next generation of goods. Rows correspond to sectors; and columns to commodities. (This is only one of the many particulars in which SFEcon's input/output format differs from Professor Leontief's: because things measured in the same physical unit are usually arranged in columns in order to visualize their totals, we have data for a given physical commodity organized in columns, while rows organize the data for a given sector.)

- 'Prices & Values' presents the values of marginal product occurring at each cell of the I/O structure. Row 0 contains the marginal costs of producing the commodities J. A horizontal vector across the top of this matrix exhibits current prices.

Initialize the apparatus by clicking the RESET button. TIME should show as zero. The three model economies are now in their initial state, where they are identical and therefore have no impulse to trade with one another. Adjust the SLIDER until 5 years are showing in the SIMULATE UNTIL box. Click START. Pressing the RESET button will have returned the model to its initial general equilibrium; so this emulation merely confirms stability for the optimal state. The emulator should mark time's passage by nothing more than a bit of twitching in the last digits of some of the larger numbers. When the emulation terminates, TIME should be at 5 and SIMULATE UNTIL should be at 10.

When you are ready to proceed, adjust the slider to its minimum. This constrains the model to emulate a single year before stopping, so SIMULATE UNTIL should be at 6. Now click the STIMULATE button and note that TIME has been reset to 0, SIMULATE UNTIL is now 1, and the numbers in the matrices express a chaotic state in which none of the marginal values equals its corresponding price. Press the START button to advance the emulation by one year and observe that the numbers in the matrices bounce around yet again.

Each press of START thereafter presents a new chaotic state until approximately year 5 when the initial stimuli are mostly absorbed. Repeated clicks of START will now advance the asymptotic approach of the model to the new general optimum implicit in its parameters. Note that the model does not 'know' where this optimum lies, and will not 'know' when the new optimum is eventually embodied in the model's state variables. All these engineering-dynamic models ever 'know' is their current state and their rules for computing their next state. Whether or not the next state happens to recapitulate the current state is irrelevant to their operation.

At some point the experimenter will likely wish to advance the emulation to a precise Pareto optimum. This is accomplished by moving the slider down to its limit, which will have SIMULATE UNTIL equaling TIME plus 50 years. Clicking on START will emulate each of those years, reporting the system's current state as it goes. Note that fewer and fewer numbers change with each iteration, and that the changes become smaller and smaller until a steady-state is in evidence. This will be the same steady-state as the one in which the model arrived.

Physical exchanges are now steady because the marginal value of each commodity J in each sector I equals commodity J's price, and the commodity's price equals its marginal cost of its production. As will be further elucidated below, this state of general optimality will have arrived just as all commodities' supplies come into alignment with their demands, and all markets clear.

III. Stasis

8. Physical Equilibrium

The specific numerical contents of our demonstration workbook in its 'as delivered' form are arrived-at by extending the 'Initial Experiment' above for a period of several decades. Those numerical specifics will be referred to in the following discussions.

Once again, the physical input/output matrix R_{IJK} presents the physical replenishment rates at which the sectors I of an economy K are taking the commodities J off the market in order to replace the assets currently being used-up in creating the next generation of goods. Rows correspond to sectors; and columns to commodities. Figure 8-1 highlights these data for column J=3, representing the non-durable good in economy K=1.

Economy 1's export/import profile is presented as a horizontal auxiliary vector above the physical input/output matrix. This vector introduces SFEcon's sign convention: things going into the matrix are presented as positive quanta to suggest increasing distance from the observer; that which comes out of the matrix are negative quanta, indicating decreasing distance from the observer. Thus exports are indicated as positive, while imports are signed in the negative. Economy K=1 is a net exporter of 110 physical units per year of the non-durable good J=3.

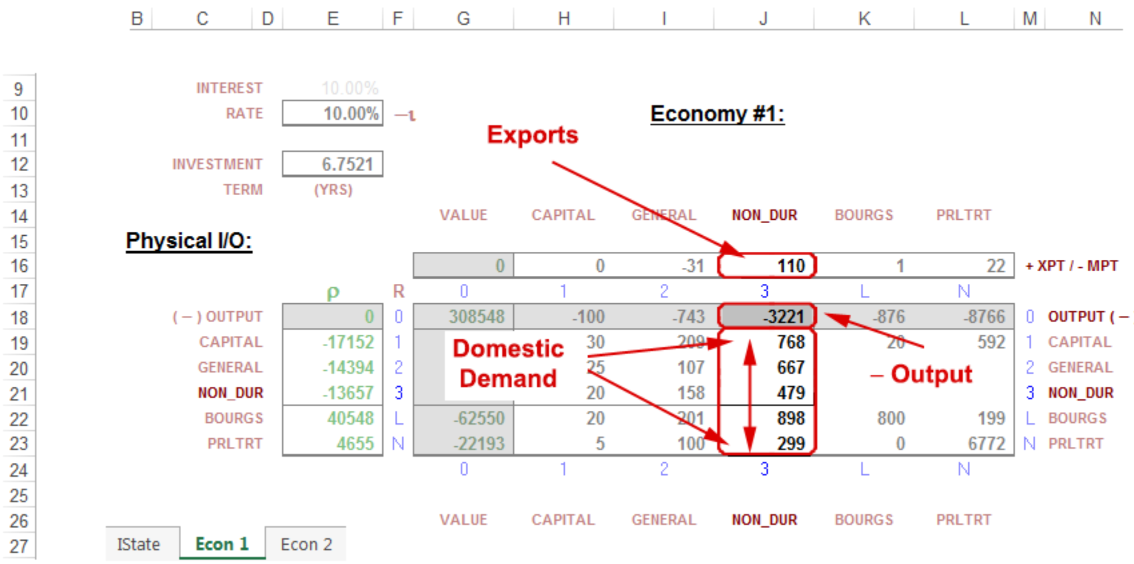


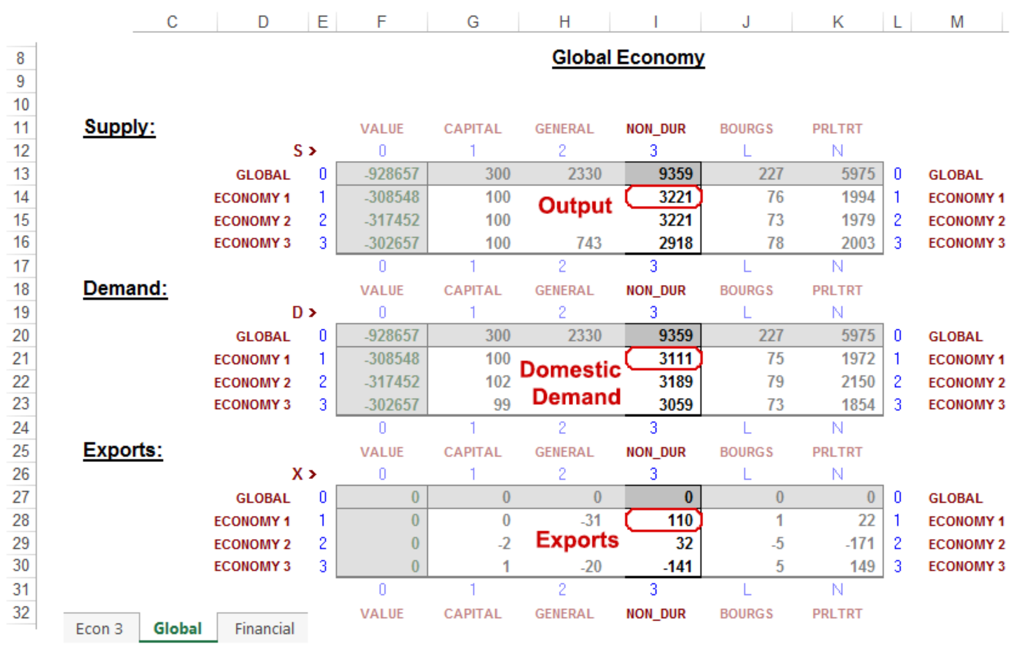
Figure 8-1: Physical Equilibrium in One Economy

Economy 1's production of the non-durable good is found in Row 0 of the I/O matrix. The –3221 at cell R_{031} indicates that economy 1 is producing the non-durable good at a rate of 3221 physical units per year. Outputs Y are presented in the negative because production is something 'coming back to us' out of economic activity, as opposed to the rates at which assets are exhausted in producing the next generation of goods – which are presented as positive quanta in the body of the matrix. Sectors exhibit their physical output rates Y in Row 0 of the I/O matrix at the column corresponding to their row index.

Elements of domestic demand D_{JK} for non-durable goods are distributed among the row elements of column J=3 in R_{IJK}. These quanta represent the physical rates at which the sectors I=1…N are taking commodity J=3 off the market to replenish asset stocks being depleted by production. Since we are in a demonstrable steady state, we may presume these quanta are also the rates at which the non-durable good is being consumed in the productive processes of the sectors I. And, having proclaimed this state to be optimal because it is steady, we can further presume that these takings from the market are elements of demand.

Economy 1 presents good 3 in physical stasis because all the highlighted quanta in Figure 8-1 add to zero. Since these data reside in a spreadsheet, this result can be easily verified for all goods J in all economies K. These results are brought together on the <u>GLOBAL</u> worksheet shown in Figure 8-2, where we find tabulations of all the model's data respecting production, demand, and exports.

Figure 8-2 highlights the data discussed in relation to the non-durable commodity J=3 in economy K=1. By extension, an experimenter should be able locate the upshot of all analyses of the sort summarized in Figure 8-1 for any commodity J in any economy K. Note that the <u>GLOBAL</u> worksheet is one of the experiment's portals: it permits observation of global physical imbalances gradually forming themselves into a universal resolution of supply with demand. Whenever the model subsides to a steady state, domestic output is just sufficient to meet domestic demand, plus exports, minus imports, for all J,K.



Global Economy

Supply:

S >		VALUE 0	CAPITAL 1	GENERAL 2	NON_DUR 3	BOURGS L	PRLTRT N		
GLOBAL	0	-928657	300	2330	9359	227	5975	0	GLOBAL
ECONOMY 1	1	-308548	100	Output	3221	76	1994	1	ECONOMY 1
ECONOMY 2	2	-317452	100		3221	73	1979	2	ECONOMY 2
ECONOMY 3	3	-302657	100	743	2918	78	2003	3	ECONOMY 3

Demand:

D >		VALUE 0	CAPITAL 1	GENERAL 2	NON_DUR 3	BOURGS L	PRLTRT N		
GLOBAL	0	-928657	300	2330	9359	227	5975	0	GLOBAL
ECONOMY 1	1	-308548	100	Domestic	3111	75	1972	1	ECONOMY 1
ECONOMY 2	2	-317452	102	Demand	3189	79	2150	2	ECONOMY 2
ECONOMY 3	3	-302657	99		3059	73	1854	3	ECONOMY 3

Exports:

X >		VALUE 0	CAPITAL 1	GENERAL 2	NON_DUR 3	BOURGS L	PRLTRT N		
GLOBAL	0	0	0	0	0	0	0	0	GLOBAL
ECONOMY 1	1	0	0	-31	110	1	22	1	ECONOMY 1
ECONOMY 2	2	0	-2	Exports	32	-5	-171	2	ECONOMY 2
ECONOMY 3	3	0	1	-20	-141	5	149	3	ECONOMY 3

Figure 8-2: Global Physical Equilibrium

9. <u>Input/Output Balances</u>

Article 3 introduced the hyperbolic descriptions of production and utility tradeoffs that are essential to SFEcon modeling:

3-1) $$Y = Z \cdot \left[1 - \frac{U_1 \cdot U_2 \cdot \ldots \cdot U_N}{(U_1 + E_1)\cdot(U_2 + E_2)\cdot \ldots \cdot (U_N + E_N)} \right]$$

The [Y,E_J] set in the hyperbolic production function recapitulated above has a physical output Y created by <u>e</u>xhausting physical inputs <u>E_J</u>. In proclaiming economic stasis, we presume to be describing a state in which the physical replenishment rates R_{IJK}

developed in Article 8 must be just sufficient to offset all E_{IJK}. If E_{IJK} and R_{IJK} are to maintain cell-for-cell concurrence, then the E_{IJK}'s must eventuate in just enough production Y_{IK} to continuously re-supply everything removed from the market by the actions of R_{IJK}.

As shown in Figure 9-1, the parametric sets [Z,U_J], visualized in Figure 3-1 and referenced in calculating production rates Y_{IK}, are organized to coincide with the physical input/output matrix. The [Y,E_J] and [Z,U_J] sets for the non-durable goods sector I=3 of economy K=1 are highlighted in Figure 9-1.

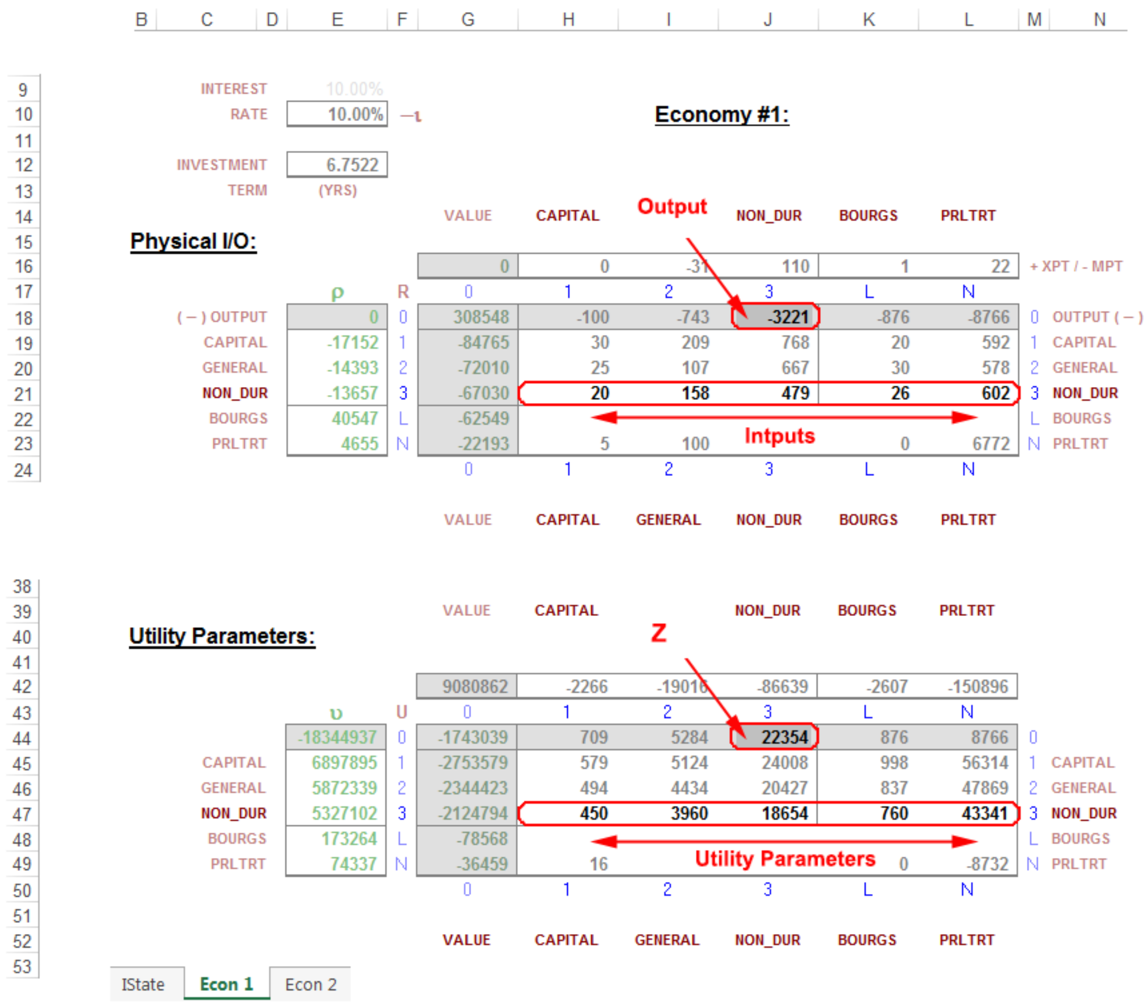


Figure 9-1: Utility and Factor Inputs

Inspection of Figure 9-1 should allow an experimenter to extract the data in Table 9-1 with which to test the validity of Equation 3-1. This equation can be verified for any of the model's three generic industrial sectors insofar as R_{IJK} and E_{IJK} are approaching stasis.

	Y	3221	Z	22354
CAPITAL	E_1	20	U_1	450
GENERAL	E_2	158	U_2	3960
NON_DUR	E_3	479	U_3	18654
BOURGS	E_4	26	U_4	760
PRLTRT	E_5	602	U_5	43341

Table 9-1: Utility and Factor Inputs for Sector 3 of Economy 1

Recapitulating the hyperbolic system's equation for marginal products,

3-2) $$\frac{\partial Y}{\partial E_J} = \frac{Z - Y}{U_J + E_J}$$

We see that the physical input/output matrix and the utility matrix are configured so that the data for computing a sector I's marginal product reside in corresponding pairs of cells. This facilitates computations of values of marginal product,

3-3) $$\pi \cdot \frac{Z - Y}{U_J + E_J} = P_J$$

with which to verify that stasis is indeed maintained in a state of Pareto optimality.

As with generic sectors, households' utility tradeoffs are expressed by the hyperbolic form in sectors I=L…N; but household utility's exposition requires the separate development to follow.

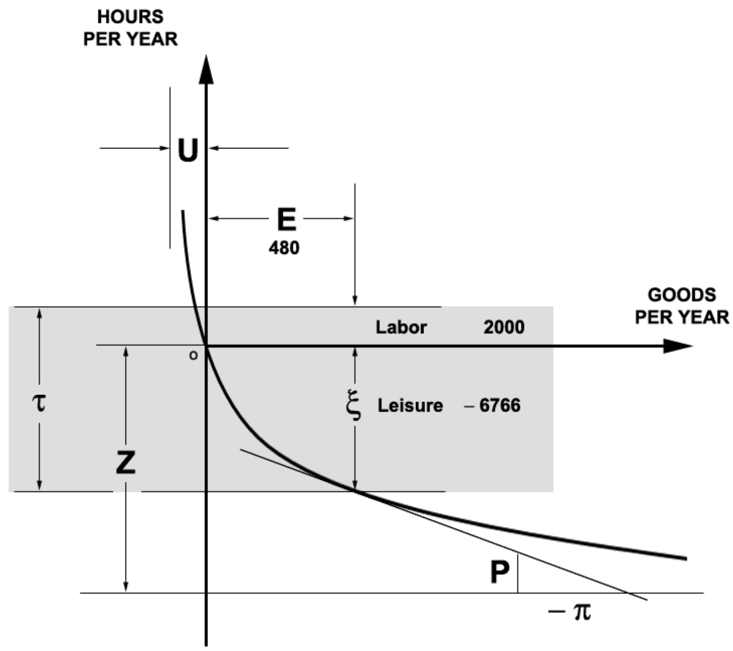
10. The Household Product

The generation of profits must of course be represented in any faithful analog to capitalism. Because SFEcon models presume to comprehend all material and financial flows, they must somehow contrive to have industrial sectors' profits received by some non-industrial sectors. Absent such a construction, there would be no possibility of completing the monetary circuit. Every SFEcon model must therefore contain at least one household sector to receive profits in the form of passive income.

Design of household sectors so as to fit with our general computational scheme for generic industrial sectors is largely a matter of re-sculpting ideas that have not changed much since Jevons.[11] Our basic premise is that households arrange their affairs for the maximization of leisure; or, more precisely, that time exhausted in the **acquisition** of things is limited by a need to reserve the time needed for the **enjoyment** of things. People generally labor in order to rest; and to earn that which provides stimulation, comfort, amusement, and security in their leisure time.

Stated formally, this means that one stops working when the enjoyment of a prospective hour of leisure is equal in value to what is earned by the last hour worked. Figure 10-1 sketches such a condition for the case of one person consuming one good.

Figure 10-1:
Household Utility



This figure arrays all the parameters developed for industrial productive tradeoffs: a set [Z,U] shapes the locus of achievable utility by locating its asymptotes; and a price environment [π,P] selects the optimal operating point [ξ,E]. The 'real wage' is represented by direct intake E of the sole consumer good. In this example, E=480 physical units/year is just sufficient to make our consumer content with $-\xi$=6766 hours/year of leisure.

Figure 10-1 introduces a parameter τ=8766 hours/year to express an inescapable limit on each consumer: there are 8766 hours in a year; and all of these hours must be accounted as either labor or leisure. Labor is therefore the residual of τ with ξ: a typical person works about 2000 hours/year, which is τ (= 8766) + ξ (= –6766). Improving economic conditions, allowing the real wage E to rise, will eventuate in greater leisure and, hence, less labor going to market.

[11] Jevons, W. Stanley: The Theory of Political Economy (1871) (Reprints of the Economic Classics). New York: Augustus M. Kelly, 1965

SFEcon's sign convention is further exercised in Figure10-1. Labor is positive because it goes into the economy for the sake of producing other things that come back out of the economy. Leisure is negative because it is one of these products. Households work to support the consumption needed for contentment within the leisure segment $-\xi$ of their continuing experience of time τ. Figure 10-1 inverts the hyperbolic utility surface's industrial representation in Figure 3-1 by making the utility parameter Z a negative quantity.

Leisure's money price π is seen operating in the negative relative to the consumable's price P because leisure is a negative quantity: the more one is at rest, the less one earns. According to the premises stated for this analysis (i.e.: the first hour of leisure is equal in value to the last hour of labor) negative π is known because positive π must be the money wage. As $|\pi|$ rises in comparison to commodity prices P, a household can afford to work a bit less and yet consume a bit more; and this marginal increase in consumption will furnish the corresponding increase in leisure.

Resolving households' operating particulars into SFEcon's input/output structure is challenging because the household product is not the singular datum 'labor' of more familiar analysis. Labor is now the residual of two items, τ and ξ; and both require their separate representations. Households' total experience of time τ resembles industrial output Y in that it creates all there is of the household product. Leisure $-\xi$ also resembles industrial output Y in that it is generated by transforming the intake of households' 'factors of production', i.e.: consumption. And labor, $\tau + \xi$, resembles industrial output Y because it is what a sector gets paid for producing.

SFEcon's resolution of these considerations has required expansion of the familiar input/output concept to include the household sectors as part of the structural whole. This creates additional diagonal elements in the matrix where the desire for leisure $-\xi$ might be given its logical expression. Figure 10-2 highlights the physical input/output pattern for the proletarian labor sector N.

Sectors' demands on the proletariat's output of time τ are distributed in column N. Where these demands issue from sectors other than N, they are understood as the employment of labor. Sector N's 'demand' for its own product, i.e. respite from labor through the 'production' of leisure, $-\xi=6772$, occurs at cell [N,N].

Matrix conventions for industrial sectors are carried through in that leisure, being computed as a negative quantity, is negated again because it is among the things that the economy produces, all of which enter the matrix structure as negative quanta. Diagonal entries for the household sector are therefore positive quanta. These conventions are also apparent in row 0, where the proletariat's total experience of time $\tau=8766$ is expressed at cell [0,N] in the negative, as would be the case for a generic industrial output.

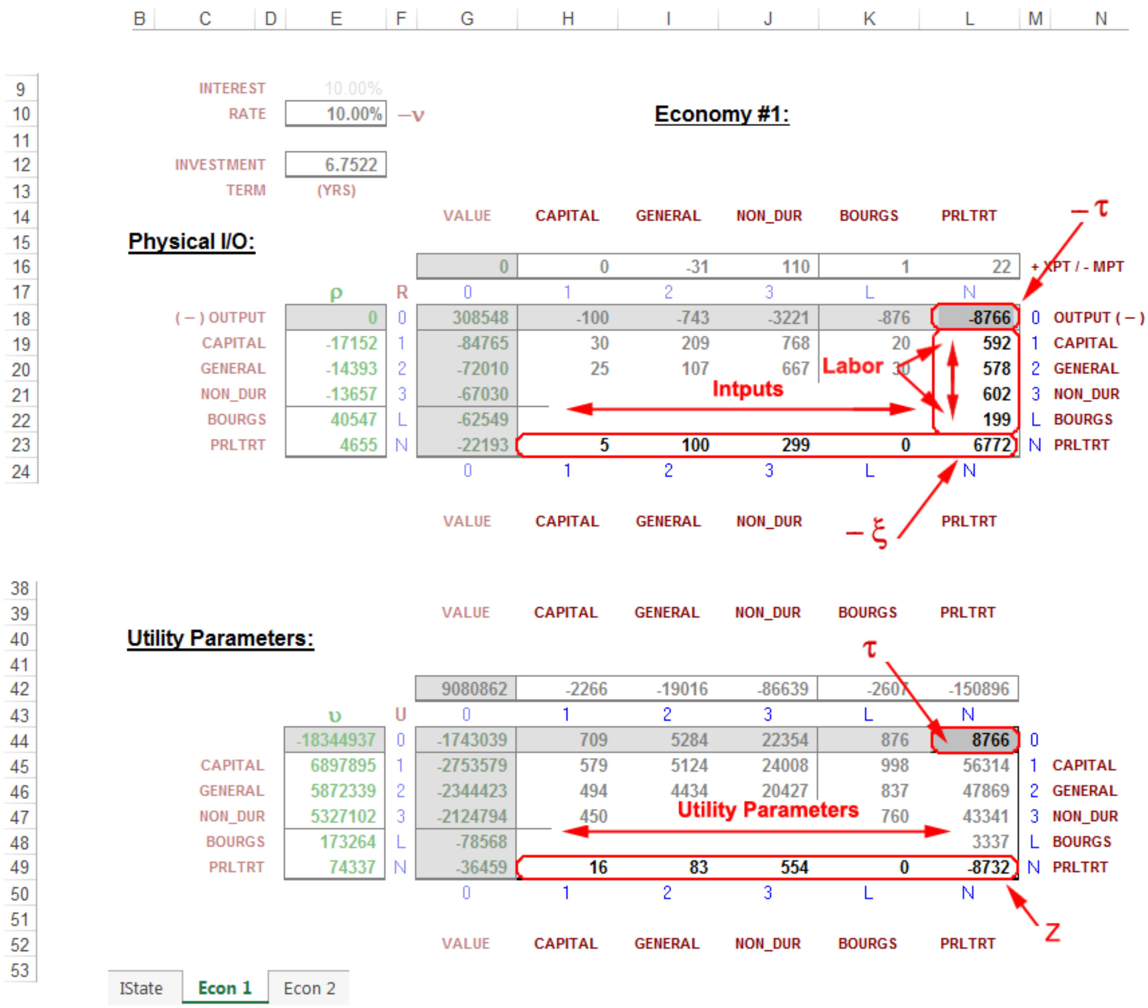


Figure 10-2: Households' Placement in SFEcon's Matrix Structure

The disposition of the proletarian labor sector's utility parameters is highlighted in the array at the bottom of Figure 10-2. Once again, proletarian labor follows conventions established for the industrial sectors, with necessary exceptions occurring at cells [0,N] and [N,N]. We note that the parameter τ is given its expression in row 0 of the utility matrix, opposite its negation at that same element of the physical I/O matrix. Households are therefore peculiar in that $U_{0LK} + E_{0LK} = 0$, which is a way of expressing that the notion of marginal product is a matter quite apart from time's inexorable unity.

A household sector's 'Z parameter' is logically placed at cell [N,N], which corresponds to the placement of household leisure $-\xi$=6772 in the physical input/output matrix. In all, matrix position [N,N] for household sectors corresponds to matrix position [0,N] for industrial sectors, where Z and –Y are placed for reference by Equations 3-2 and 3-3.

Section III's exposition of macroeconomic stasis closes with an important task left to be accomplished in Section VIII, where households will be further examined with respect to certain remaining exceptions that are necessary to accommodate household sectors within a general matrix formulation. Here we will find households' 'Z parameter' recast as a variable.

IV. Dynamics

11. Physical State Variables

Section III's portrayal of macroeconomic stasis is quite likely unique among systems of neoclassical causality for its being both general and quantifiable. Moving on to propose a marginalist dynamic for economic adjustment offers no comparable prospect for uniqueness. When time becomes a variable in the structure of a complicated nonlinear system, the possibilities for alternate structures multiply without limit.

Establishment of a definitive dynamic for macroeconomics being impossible, our ambitions from this point will be limited to establishing the most primitive structure with which to falsify economics' comprehensive finding that . . .

> *. . . neoclassicism cannot demonstrate that equilibrium would emerge as a natural consequence of agents' instrumentally rational choices*.[12]

Prices will be considered known and available to guide the adjustment of physical asset levels throughout this section. Price determination itself is necessarily deferred until its references are established – these being the model's physical state variables, along with the financial state variables arising from physical transactions at current prices.

Our discussion of equilibrium has been under obvious strains to reference these dynamics without actually giving place to them. We can now begin to relieve this tension by formally distinguishing between 1) the rates $\underline{R}_{IJK}$ at which sector I of economy K replenishes its stock of commodity J, and 2) the rates $\underline{E}_{IJK}$ at which commodity J is employed in creating the output of sector I in economy K.

Model 0's portrait of multi-sectoral interdependence has been structured on an input/output matrix of physical assets giving up their economic lives in creating the next generation of goods. And these assets are presumably being worked-off or built-up in search of optimal operating conditions. Figure 11-1 illustrates the continuous nature of these interchanges. In what can only be a small isolated economy, sector J is a public utility responsible for a waterway. The output Y_J of this sector is flow through a canal

[12] Sonnenschein, Hugo F.: "Do Walras' identity and continuity characterize the class of community excess demand functions?" *Journal of Economic Theory* vol. 6, 1973, no. 4, 345-354

(portrayed here in cross-section) that is diverted along the way for the use of other sectors that might employ the flow for navigation, irrigation, power generation, etc.

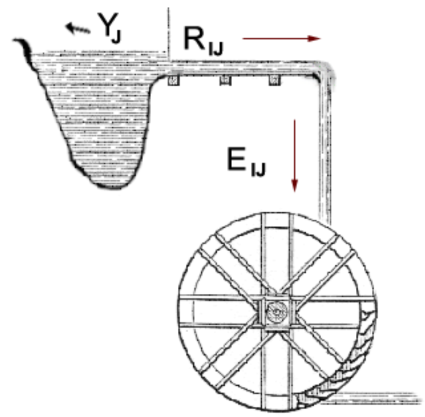


Figure 11-1: Replenishment versus Employment

One such sector I is a mill driven by a waterwheel. R_{IJ} is the rate at which sector I takes good J off the market and into its own asset stock, i.e.: into a millpond where the flow's potential energy builds up before engaging the wheel.

Our analysis to this point has treated a physical equilibrium in which the employment matrix E_{IJK} is sector IK's direct application of good J to the creation of its product. The possibility of disequilibriated physical states opens with a distinction between the rate R_{IJK} at which IK acquires J, and E_{IJK}, the rate at which J is flowing through the production processes of IK.

Implicit in this view is a presumption that economic goods, once produced, immediately enter into a sort of 'gravitational field' that pulls the economic potential out of them at rates characteristic of a commodity's logistical identity, irrespective of the productive process in which the good is employed, or whether it is employed at all. The topography in which the waterway/waterwheel of Figure 11-1 is set would, for example, dictate the ultimate rates of fluid flow through all the industries served by commodity J.

This view is reinforced by our sectoral analysis context which would have it that, once acquired, an asset must give up its useful life in producing the output of the sector that purchased it. Firms can sell-off assets; but these are generally purchased by other firms in the same sector. Since sectors only profit from the sale of their own good, sunk costs can only be recovered by exhausting existing assets in production.

These presumptions call forth a parameter V_J to distinguish the logistical identities of the goods J. V_J represents a good's physical durability in terms of the accountant's turnover fraction, 1/year. It embodies the perishability of food stuffs, the economic order quantity of an inventoried item, the tendency for consumer items to go out of style, or for heavy equipment to wear out, become technically obsolete, or require maintenance. Sectors' interest in minimizing their asset levels operates to keep turnover rates V_J pegged at the maximal, hence constant, value consistent with the asset's physical make-up. The matrix of physical assets can therefore be envisioned as the state variables actuating higher-ordered delays governed by V_J.

Figure 11-2 pictures a third-order delay in which a rate R is delayed by passing through a series of state variables A, B, and C.

Taking $\underline{R}_{IJK}$ as analogous to replenishing assets being used up, the flow of material through the delay pipeline is analogous to assets giving up their useful economic lives in creating the next generation of goods.

$\underline{E}_{IJK}$ is equated to the average rate $V_J \cdot (A_{IJK}+B_{IJK}+C_{IJK})$ at which assets are passing through the delay structure. As assets pass along the dimension of time toward exhaustion, their average rates of flow provide independent variables for the production function of Equation 3-1, contributing to a sector IKs' output Y_{IK} rather as the flow of water in Figure 11-1 'turns the wheels of industry'.

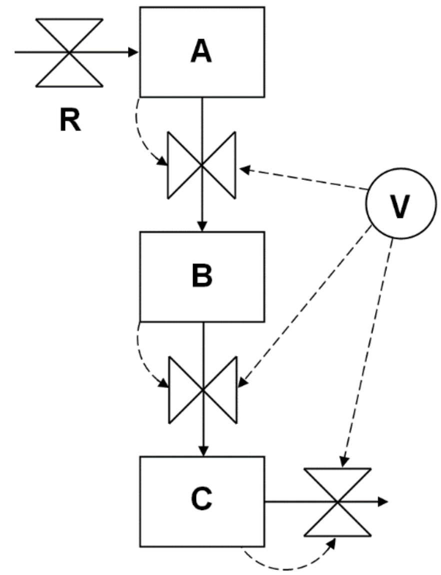


Figure 11-2:
A Third-Order Delay

Having distinguished a rates matrix R_{IJK} (the acquisition of assets to replace those that are passing out of existence) from an employments matrix E_{IJK} (assets giving up their useful economic lives in creating the next generation of goods) we can now formally 'close the loop' on the cycle of economic adjustment. Figure 11-3 presents a highly stylized [13] signal path of SFEcon's physical dynamics for a single economy K that is isolated from foreign trade.

Physical assets are called into being by output rates Y_J at the top of this figure. The oblong shape of the Y_J symbol is to indicate a vector comprising output rates for all commodities J. The flow of all commodities is indicated by a single line from a source symbol to the market vector $\underline{O}_J$ at the figure's left. Goods on the market have been produced, but are not yet contributing to the production of other goods. Conservation of mass requires that elements of the market vector be kept at or above zero.

Physical flows of goods around the circuit of Figure 11-3 must be imagined to divide once again as asset replenishment rates $\underline{R}_{IJ}$ distribute the newly-created commodities J among the sectors I. This rates symbol has been stylized into a square shape to indicate that it is a matrix, each cell of which initiates a flow into the corresponding cells of a group of higher ordered delays.

This circuit of physical flows is completed as the rates E_{IJ} at which assets give up their economic lives interact with utility parameters U_{IJ} to create the flows of output Y_J per Equation 3-1. Cell-for-cell equality between E_{IJ} and R_{IJ} thus becomes a criterion of equilibrium for economic dynamics.

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[13] and incomplete: money flows are absent, so only a potion of the model's information flows can be represented.

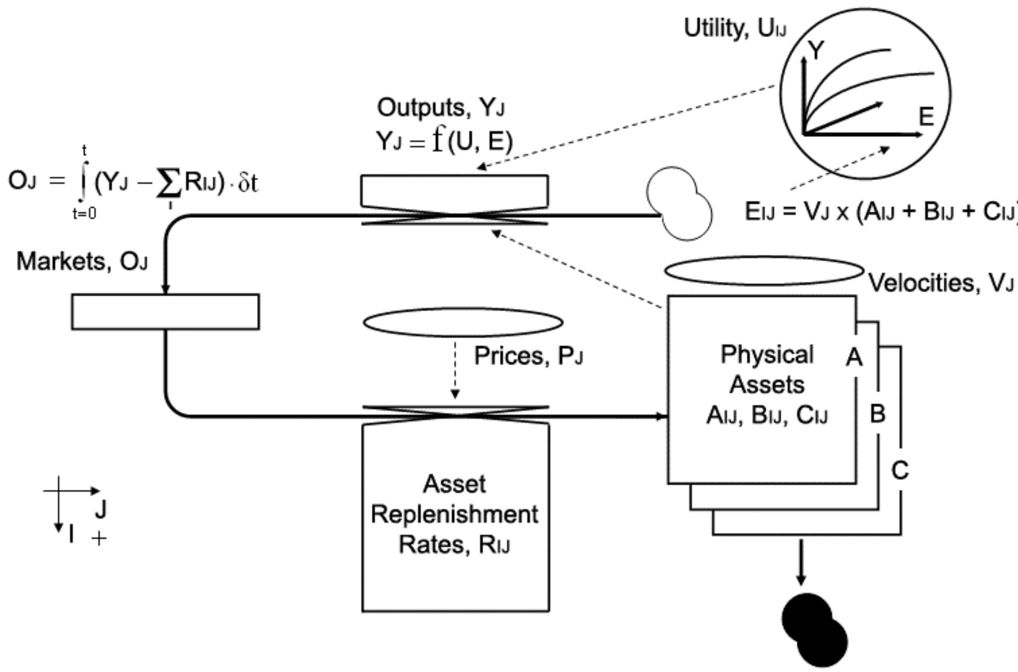

Figure 11-3: Physical Rates and Levels

Having formally distinguished E_{IJ} and R_{IJ}, we can now return to clarify certain ambiguities lingering from earlier discussions occurring in an equilibrium context where R_{IJK} and E_{IJK} were presumed identical. As indicated in Figure 11-3, it is most consistent to place $-Y_{JK}$ in Row I=0 of the rates matrix R_{0JK}. E_{0JK} then becomes a higher-ordered delay on output, which therefore embodies a flow of the entire quantity of a good J that has been produced but not yet <u>e</u>xpended.

With E_{IJK} so structured, wastage from Figure 11-3's market vector becomes simply a matter of inference. For an isolated economy with no exports or imports, the unproductive deterioration of unsold goods J is simply a negative sum on the corresponding column of E_{IJ}: $-E_{0J}$ is the totality of J that is flowing toward <u>e</u>xtinction; the sum of $\underline{E}_{IJ}$ over I is the flow of J <u>e</u>xhausting itself in the production of goods I, and the residual of their sum must be the flow of J that is being wasted. Generalizing further to an economy K having an e<u>x</u>port/import profile $\underline{X}_{JK}$ (+ for exports, – for imports) we can define K's current quantity of J <u>o</u>n the market $\underline{O}_{JK}$ through the following equation:

11-1) $$V_J \cdot O_{JK} = -\sum_{I=0}^{N} E_{IJK} - X_{JK}$$

12. Control of the Economy's Physical State

Any given system of global production and utility tradeoffs U_{IJK} offers an unlimited number of possibilities for matrices of physical employments E_{IJK} constituting physical equilibria, which we define as having just enough of every commodity produced so as to replace that which is expended in creating the next generation of goods. The criterion of physical equilibrium is, therefore, inadequate to control the dynamics of continuous asset readjustment: any system with the freedom to choose among equally attractive physical states will almost certainly resonate among these states.

Having set forth Figure 11-3 as the irreducible dynamic of a macroeconomic system, we have reduced the economic theorist's task to a determination of asset replenishment R_{IJK} – other elements of Figure 11-3 being matters of definition only. A minimal requisite for stability in this system would have R_{IJK}'s tending toward a unique state such as general optimality. Our determination of R_{IJK} must therefore exhibit critical references to the commodity prices P_{JK} with which optimality is defined.

Article 3's treatment of polynomial factoring epitomized the optimal functioning of an economic sector in terms of 'financial discriminant' ζ_{IK} expressing the interaction of a sector's utility tradeoffs [Z,U] with the current price environment [π,P]:

3-8) $$\zeta = (\pi \cdot Z \cdot P_1 \cdot U_1 \cdot P_2 \cdot U_2 \cdot \ldots \cdot P_N \cdot U_N)^{\frac{1}{N+1}}$$

Equation 3-4 shows that in an optimal economic state, the optimal values of [Y,E] (hereafter designated [ξ,Q]) are disclosed in mathematically closed-form if ζ is known:

12-1) $$\begin{aligned} \zeta &= \pi \cdot (Z - \xi) \\ &= P_1 \cdot (U_1 + Q_1) \\ &= P_2 \cdot (U_2 + Q_2) \\ &\vdots \\ &= P_N \cdot (U_N + Q_N) \end{aligned}$$

The optimal rate Q_{IJK} for IK to use J can be calculated by rearranging the Equations 12-1 . . .

12-2) $$Q_{IJK} = \frac{\zeta_{IK}}{P_{JK}} - U_{IJK}$$

. . . while deriving Equation 12-2's ζ_{IK} from the first of Equations 12-1:

12-3) $$\zeta_{IK} = \pi_{IK} \cdot (Z_{IK} - Y_{IK})$$

Here we assert that a sector IK's financial discriminant ζ references the current price π of its product, and its current state, as embodied in its current rate of output Y (which has been substituted for ξ to create Equation 12-3). The validity of this assertion is based on its observed efficiency in directing SFEcon's emulators.

Figure 12-1 summarizes our case for positive control of the economic state by a tendency toward economic optimality. One limb of a production function has been isolated to distinguish between actual rates of asset usage E and optimal rates Q in terms of prices [π,P] and the value of marginal product **VMP**.

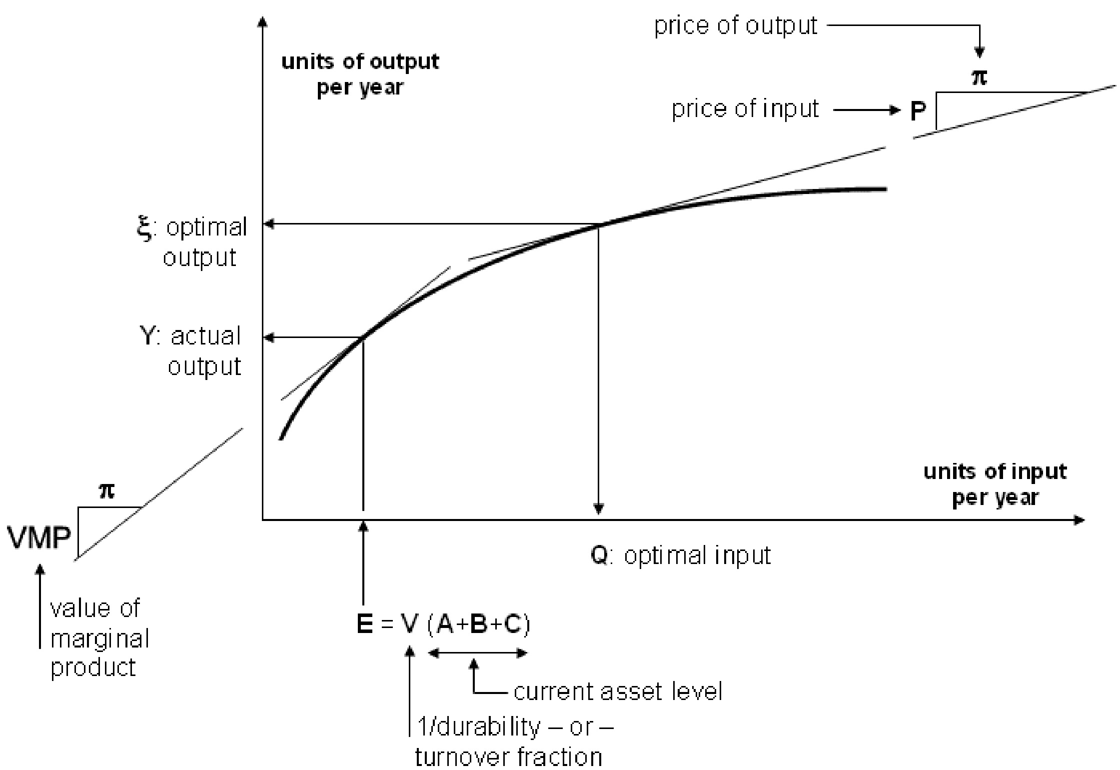

Figure 12-1: Control of the Economic State

The most direct strategy for aligning E_{IJK} with Q_{IJK} is to set the replenishment rate R_{IJK} at the optimal rate Q_{IJK} of asset expenditure. (We pause to note that R_{IJK} can only equal Q_{IJK} insofar as supplies are sufficient to meet all demands, and that some regime of allocation must govern asset replenishment when supplies are deficient. Model 0's allocation regime will be discussed shortly in conjunction with foreign trade.) These logistics have E_{IJK} driven toward Q_{IJK} most quickly for volatile assets characterized by high values of V_J; while assets that are durable, hence expensive, will proceed toward optimal levels at more conservative rates dictated by lower V_J's.

Forcing E into equality with Q drives **VMP** toward identification with P irrespective of the final point at which the adjustment process settles; and the process is being driven by a criterion having no need of (nor place for) prior knowledge as to the final settlement point. Productive optimality at $Y=\xi$ is stable because variations from it induce dis-economies (a gap between an input's price and its **VMP**) that are only resolved by optimality's reinstatement at the same or another point. And it matters not whether variation was introduced by logistical miscalculation or adjustments to prices dictated by changes in the shape of another production function elsewhere in the system.

13. Global Supply and Demand

Article 11 culminated in a definition of O_{JK}, i.e., the level of a commodity J that has been produced in an economy K, but which remains un-purchased and therefore wasting its economic potential at a rate $V_J \cdot O_{JK}$, where V_J is J's turnover fraction. Our notion of supply S_{JK} is defined by adding this 'market pressure' to the rate Y_{JK} at which J is currently produced in K:

13-1)
$$S_{JK} = Y_{JK} + V_J \cdot O_{JK}$$

Article 12 identified an individual sector IK's demand Q_{IJK} for commodity J via Equation 12-2. Our notion of the domestic demand D_{JK} sums demands of the sectors I for J in K:

13-2)
$$D_{JK} = \sum_{I=1}^{N} Q_{IJK}$$

Simple addition across all M of the economies K provides global totals Y_{J0}, O_{J0}, S_{J0}, and D_{J0} for all the variables subscripted JK in Equations 13-1 and 13-2. Where S_{J0} and D_{J0} are equal for given commodity J, we can be assured that 1) there exists an export/import profile X_{JK} such that X_{J0} is null, and 2) all domestic demands D_{JK} are satisfied by the difference between domestic supply S_{JK} and exports X_{JK}:

13-3)
$$D_{JK} = S_{JK} - X_{JK}$$

(Per the SFEcon sign convention, a negative value for X_{JK} indicates a quantity that is imported.)

Working as we are within the neoclassical paradigm, we can anticipate creating a regime of prices that tends to equate demand with supply; but this tendency is necessarily imperfect for any dynamic model, as indeed it must be for any objective counterpart to what the model portrays. It is therefore essential to exhibit at least one mechanism whereby the X_{JK}'s are calculated such that X_{J0} will 1) always approximate zero to the extent that the integer zero can be expressed as a real number by a digital computer, and 2) vary from zero in a way that is temporally unbiased to the positive or negative.

This is most easily done by envisioning a quantity D_J' for the amount of global demand D_{J0} that will be satisfied at any given point in time, and a quantity S_J' for the amount of global supply S_{J0} that will be used at any given point in time. At any moment, one of two things must be happening: S_{J0} will be diminished to S_J'; or D_{J0} will be diminished to D_J'. The factors accomplishing these diminutions are named FS_J and FD_J; and, at any given moment, one of these factors must unity, while the other must be less than unity.

The factors FS_J and FD_J both derive from the ratio F_J of global supply S_{J0} to global demand D_{J0}:

13-4) $$F_J = S_{J0} / D_{J0}$$

13-5) $$\text{if} \quad F_J \geq 1 \quad \text{then} \quad FD_J = 1 \quad \text{and} \quad FS_J = 1/F_J$$

13-6) $$\text{if} \quad F_J < 1 \quad \text{then} \quad FS_J = 1 \quad \text{and} \quad FD_J = F_J$$

FS_J and FD_J determine e<u>x</u>ports/imports <u>X</u>$_{JK}$ of commodity J from/to economy K per Equation 13-7:

13-7) $$X_{JK} = FS_J \cdot S_{JK} - FD_J \cdot D_{JK}$$

Export/import profiles are reported through the <u>GLOBAL</u> portal as shown in Figure 13-1. This display verifies that the overall global system never attempts to export more than is imported, or import more than is exported, even while Equation 13-3 remains out of balance until the model enters stasis.

Exports:

	X >	VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT		
		0	1	2	3	L	N		
GLOBAL	0	0	0	0	0	0	0	0	GLOBAL
ECONOMY 1	1	0	0	-31	110	1	22	1	ECONOMY 1
ECONOMY 2	2	0	-2	52	32	-5	-171	2	ECONOMY 2
ECONOMY 3	3	0	1	-20	-141	5	149	3	ECONOMY 3
		0	1	2	3	L	N		
		VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT		

Econ 3 | Global | Fina

Figure 13-1: Global Export/Import Profiles

FD_J also serves to prevent the sectors IK from taking J off the market in excess of what is being supplied:

13-8)
$$R_{IJK} = FD_J \cdot Q_{IJK}$$

Here we see that, whenever the global supply of J equals or exceeds global demand and FD_J is unity, the rate R_{IJK} at which a sector IK takes a commodity J off the market (per Figure 11-3) equates to the optimal rate Q_{IJK} for IK to use J (per Equation 12-2). When supplies of J are not adequate to meet demand, FD_J less-than-unity rations asset replenishment rates. An overabundance of supply for J will be absorbed into the market, where (as we shall see) it depresses prices.

The full matrix of goods on the market O_{JK} is continuously reported on the GLOBAL sheet. This display is obviously only informative in showing that Model 0's markets are kept almost perfectly clear even while the model proceeds through some complicated dynamics. Small negative quanta appearing here and there on the market are allowed as indicia to the model that a given commodity J must be rationed. 'Conservation of mass' is required insofar as a market is only allowed to go negative to the extent possible during one differential element of time, given that the market variable O_{JK} began that differential element of time as a positive quantity. Conservation of mass is reinstated by the definition of supply S_{JK} as output Y_{JK} plus market pressure $V_J \cdot O_{JK}$ in Equation 11-1: where market pressure is a tad negative, supply becomes a tad less than current output; and demands are never allowed to be satisfied in excess of available supply.

14. Optimality and the Reference Commodity

Our inquiry into prices and values begins with knowledge of the current level $A_{IJK}+B_{IJK}+C_{IJK}$ of every commodity J in the inventories of every sector I in every economy K. These variables are defined as being controlled by replenishment rates R_{IJK} and employment rates E_{IJK}, where E_{IJK} is defined by $V_J \cdot (A_{IJK}+B_{IJK}+C_{IJK})$ and R_{IJK} must remain undefined until prices are known.

To proceed from this point we must first establish assurances that money prices will express the commodity's intrinsic values with respect to the economy's underlying shape of production and utility tradeoffs. Toward this end, every SFEcon model transacts a zeroth commodity through a zeroth sector. Good $J=0$ constitutes an artificial financial unit, or 'reference commodity', which we offer as an approach to the elusive notion of absolute value.

It might be said that SFEcon's valuation of commodities in terms of good 0 aspires to the operational properties Piero Sraffa ordained for his reference commodity.[14] But, where Sraffa is referring to the economy's operation at full capacity, SFEcon refers to the economy's operation at the general optimum implicit in its parameters.

Our rather Alexandrine solution to the Gordian problem of value does not presume to measure value, but ordains 'value' as a unit of measure that the economist might use to gauge economic activity in a manner consistent with the familiar engineering disciplines. Such praxis is instantiated by the thermodynamicist who does not presume to know what 'energy' is, but whose science makes critical references to a fixed zeroth energy state and to a constant energy unit. Zero can be anywhere, and the unit can be of any size; but these references must not be disturbed by the operations of theory, and their invariant nature must be confirmed in experimental results.

The parallel task for economics would be to select a reference currency, say the 1997 US dollar; measure all global output during 1997 in that currency unit; and install this arbitrary rate of value's flow as a constant of theory. SFEcon establishes the location of 'zero' and the size of 'one' in an artificial flow of value induced by a constant output rate Y_{00} for the zeroth good in the zeroth (i.e. global) economy during economic year zero (e.g. 1997). Our good 0 is denominated in what we call a global currency unit, the GCU or ~~G~~, as suggested by its uses for computing economies' currency values relative to one another, as well as the values that a currency might have relative to itself at another point in time.

[14] Sraffa, Piero: *Production of Commodities by Means of Commodities, Prelude to a Critique of Economic Theory*. Cambridge: Cambridge University Press, 1960

Y_{00}'s initial magnitude, in Ǥ/yr., is given by the value of all physical asset flows E_{IJK} as they exist in the model's initial stasis. The currency unit used for this valuation is, once again, arbitrary; but the initial global commodity values Ψ_J Ǥ/unit must exhibit relative magnitudes that satisfy a general price equilibrium. In Model 0's initial (and any subsequent) stasis, the economic sectors IK are valued by the sum of their physical asset turnovers E_{IJK}, weighted by commodity values Ψ_J:

14-1) $$E_{I0K} = -\sum_{J=1}^{N}\left(\Psi_J \cdot E_{IJK}\right)$$

Model 0 is initialized by setting value's constant global flow rate Y_{00} at the sum of the E_{I0K} variables over all IK. Thereafter, all of Y_{00} must be kept coursing through the E_{I0K}'s, in their 'proper' proportions, via the model of Figure 11-3. The mechanism by which these proportions are kept 'proper' will be elucidated in Article 24.

Tests of the valuation regime to be based on this structure will be operative. If commodity values Ψ_J are to be absolute, then they must reassert themselves whenever the economic system re-enters stasis after a period of economic adjustment – provided that the stimuli to which the economy adjusts do not involve changing the shapes of production and utility tradeoffs. Simulations of Model 0 based on these 'elastic stimuli' (exogenously changing the amount of some good's quantity somewhere in the model) must also return all E_{I0K}'s to their initial values, just as they exactly replicate all elements of E_{IJK}.

Adjustments based on 'plastic stimuli' (exogenously changing the shape of any production tradeoff) must change all commodity values. Plastic stimuli eventuate in all elements of E_{IJK}, including the E_{I0K}'s, having achieved new and stable levels. The new stasis must also exhibit changes to all commodity values Ψ_J; these new values must express a new general price equilibrium while validating Equation 14-1; and the sum of all E_{I0K} must nonetheless remain constant at Y_{00}.

Anticipating these tests, we resolve to formulate commodity values Ψ_J on 1) the valuations E_{J0K} Ǥ/yr. of the sectors JK that produce commodity J; 2) the shape of all the technologies for producing good J; and 3) the marginal values that J has among all the sectors IK to which it is input.

15. Optimality and Marginal Value

Article 3's proposed solution to the polynomial factoring problem featured a variable ζ with which to describe the relation between a sector's technology $[Z,U_J]$ to current prices $[\pi,P_J]$ when its physical state $[Y,E_J]$ is at its optimum $[\xi,Q_J]$.

We note for continuing reference that, whether in or out of equilibrium, the system of Equations 3-4 offers an interpretation of the price vector as the current marginal value of commodity J to a sector IK.

3-4)
$$\begin{aligned} \zeta &= \pi \cdot (Z - Y) \\ &= P_1 \cdot (U_1 + E_1) \\ &= P_2 \cdot (U_2 + E_2) \\ &\vdots \\ &= P_N \cdot (U_N + E_N) \end{aligned}$$

One approach to ζ from a disequilibrium state will be named θ; it derives from a sector's budget σ for current asset replenishment. This line of mathematical development replaces ζ with θ in Equations 3-4, and then extracts N of the following equations:

15-1)
$$P_J = \frac{\theta}{U_J + E_J}$$

Multiplying both sides of Equation 15-1 by E_J creates elements of expenditure on the left side:

15-2)
$$P_J E_J = \frac{\theta E_J}{U_J + E_J}$$

Adding Equations 15-2 for all inputs J brings forth the budget σ:

15-3)
$$\sum_{J=1}^{N} P_J E_J = \sigma = \theta \sum_{J=1}^{N} \frac{E_J}{U_J + E_J}$$

Equation 15-3 can then be solved to establish θ as an expression of a sector's current asset levels and its expenditures σ for their replenishment:

15-4)
$$\theta = \sigma \Big/ \sum_{J=1}^{N} \frac{E_J}{U_J + E_J}$$

Equation15-4 can be readily particularized to a given sector IK and then restated in terms of the constant value unit GCU by replacing σ \$/yr. with a given sector IK's value of asset turnover $-E_{I0K}$ ~~G~~/yr.:

15-5)
$$\Theta_{IK} = -E_{I0K} \Big/ \sum_{J=1}^{N} \frac{E_{IJK}}{U_{IJK} + E_{IJK}}$$

Equation 15-5 has substituted Θ_{IK} ~~G~~/yr. for θ \$/yr. to indicate the translation from units of a local (hence variable) currency flow per year σ to a rate of value flow in constant units of ~~G~~/yr. based on E_{I0K}. We now have a financial discriminant Θ_{IK} that expresses every sector's current (not necessarily optimal) physical state in terms of a single unit of measure. It can be used to determine the current absolute value of marginal product by substitution for θ in Equation 15-1.

16. <u>Absolute Commodity Values</u>

Say's Law sanctions the abstraction of markets as setting a commodity J's price so as to induce demand equal to however much of J is currently being supplied. On our way to exploiting this truism it must be noted that invocations of Say are often derided as absurd because,

> *As Marx showed far better than did Keynes, the conditions under which Say's Law is correct are not those of a capitalist economy*.[15]

Even though . . .

> *Marx conceded that, if the sole motivation of exchange is consumption, then aggregate supply **is** aggregate demand* [16]

Thus we are encouraged to believe that supply creating its own demand is only an artifact of equilibrium models that are so unsophisticated financially as to render prices as mere counters in transactions that are just as well specified in terms of barter.

Here we can only acknowledge these contrary findings as we pass on to complete SFEcon's exhibition of financial control based on Say's Law prices. If (in a matter of further controversy) we might presume to know J's current demand schedule D-D, then price determination can be visualized according to Figure 16-1.

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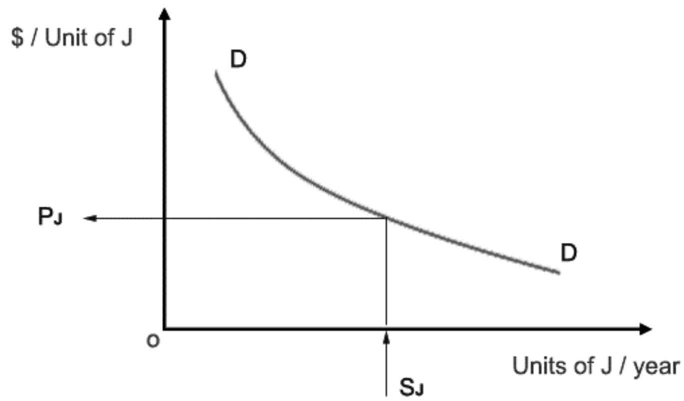
[15] Keen, Steve in Kates, Steven (Ed.): *Two Hundred Years of Say's Law; Essays on Economic Theory's Most Controversial Principal*. Northampton (MA): Elgar, 2003. P. 200

[16] Ibid, p. 201

Figure 16-1:
Say's Law Prices



The SFEcon algorithm's continuous awareness of supply rates S_J through Equation 13-1 gives us the independent variable for this analysis. Price determination then becomes a matter of establishing a dynamic demand schedule D-D that continuously re-expresses J's marginal value among the sectors that use it.

Equation 16-1 agglomerates all the Equations 15-1 for the sectors I to which a good J is input. This consensus of marginal values for commodity J is constructed by summing the numerators for all the Equations 15-1 and dividing by the sum of all denominators.

16-1)
$$P_J = \frac{\sum_{I=1}^{N} \theta_I}{\sum_{I=1}^{N} \left(U_{IJ} + E_{IJ} \right)}$$

The effect here is to create a weighted average of J's marginal values among the sectors I, where greater weight is given to sectors I having the greater 'power to command their price' as measured by θ_I. When demand D_J is substituted for the sum on E_{IJ} in this equation's denominator, we have an algebraic expression of Figure 16-1's demand schedule D-D. When supply S_J is then substituted for D_J, we arrive at an expression for J's price P_J . . .

16-2)
$$P_J = \frac{\sum_{I=1}^{N} \theta_I}{\sum_{I=1}^{N} U_{IJ} + S_J}$$

. . . that is effective insofar as Equation 15-4's definition of θ might presume to 'know' the sectors' current budgets σ.

Having resolved to approach price through the abstraction of value, we note that evaluation of a commodity J must reference the technologies of all the sectors JK worldwide. Equation 16-2's global counterpart is easily derived because its elements (θ having its value equivalent in Θ) are all measured in physical units that can be added across all M of the economies K:[17]

16-3)
$$\psi_J = \frac{\sum_{K=1}^{M}\sum_{I=1}^{N}\Theta_{IK}}{\sum_{K=1}^{M}\sum_{I=1}^{N}U_{IJK} + \sum_{K=1}^{M}S_{JK}}$$

Equation 16-3 evaluates every commodity J based on utility parameters U_{IJK} and expressions of the global economy's current physical state given by $E_{IJK}=V_J\cdot(A_{IJK}+B_{IJK}+C_{IJK})$. Commodity values Ψ_J are now completely specified except for the mechanism by which sectors' values E_{I0K} are kept in their proper proportions. Again, this specification must be put-off until Article 24.

VI. Money

17. Interest Rates

Having stated that our ambition for Model 0 is to establish the most primitive and normative view of economic adjustment, our choice (from a great many worthwhile possibilities) for an interest rate formulation will reference value rather than money. Model 0's interest rates $-\iota_K$ for the economies K are creatures of K's technology, its current state, and the global economy's current commodity values Ψ_J.

The most elementary understanding of a return rate presumes that the value of the industrial sectors' assets has been diverted from the household sectors' immediate consumption into the production of value flows that are greater than the value of what is expended in production.

[17] The alert reader will observe a tacit assertion that composite production functions for all the world's technologies producing a given commodity J can be created by simply adding production parameters U_{IJK}, and all asset expenditure rates E_{IJK}, across all economies K. Though beyond the scope of this monograph, our practice here is entirely defensible for the unique case of hyperbolic systems of technical indifference operating near their general optima. Please consult sfecon.com's article on hyperbolae for details.

Model 0 summarizes the industrial sectors' asset valuations α_K organized in column J=0 of economy K's physical state variables. Indexing the first household sector in the matrix structure as L (for labor/leisure), the industrial sectors' row indexes would range from 1 to L-1. Asset valuation is then given by:

17-1)
$$\alpha_K = -\sum_{I=1}^{L-1}\left(A_{I0K} + B_{I0K} + C_{I0K}\right)$$

As with any physical stock, 'value' is propelled by its turnover fraction V_0/yr. As discussed in Article 11, Model 0 makes due with turnover fractions that are presumed to be constant expressions of an asset's physical make-up. By analogy, V_0 is a constant 1/yr. because financial variables such as the interest rate $-\iota_K$ or the investment term T_K are always computed in reference to asset turnover that occurs **per annum**.

The value of assets continually being extinguished $V_0 \cdot \alpha_K$ ~~G~~/yr. produces a suite of outputs Y_{JK} having a composite value that exceeds cost of sales $V_0 \cdot \alpha_K$ by profits $-\iota_K \cdot \alpha_K$. These profits must, of course, be earned in respect to the value of all production that is currently available for use in creating the next generation of goods, including the values those products remaining idle on the markets. Thus we equate the value of assets being expended, plus the value of profits to be required of those expenditures, with the value of products currently in supply in order to determine the current interest rate $-\iota_K$:

17-2)
$$\sum_{J=1}^{L-1}\left(\Psi_J \cdot S_{JK}\right) = \alpha_K \cdot \left(V_0 - \iota_K\right)$$

Since commodity values Ψ_J and supply rates S_{JK} are known from Equations 16-3 and 13-1, Equation 17-2 completely determines ι_K:

17-3)
$$\iota_K = V_0 - \frac{\sum_{I=1}^{L-1}\left(\Psi_J \cdot S_{JK}\right)}{\alpha_K}$$

Here we can see the presence of goods on the ($S_{JK} > Y_{JK}$) operating to raise interest rates (i.e. increase ι_K's negative magnitude).

18. Investment

Computation of economic adjustment's driving functions R_{IJK} in Section IV presumed continuous knowledge of commodity prices P_{JK}. That presumption gives rise to a further inference of each sector IK's net cash flows $-\rho_{IK}$ \$/year,

$$\rho_{IK} = \sum_{J=1}^{N} \left(P_{JK} R_{IJK} - P_{IK} R_{JIK} \right) - P_{IK} X_{IK} \quad \text{18-1)}$$

which constitute monetary driving functions to be resolved into financial state variables that must eventually be shown to control economic adjustment.

Note the use of SFEcon's sign convention in ρ's construction: earnings are generally expressed as a negative, i.e.: costs minus income. Negative ρ's emanating from generic industrial sectors indicate profits 'coming back to us' from productive activity. The household sectors generally report positive ρ's, indicating that consumption exceeds wages – a difference that must be made up by passive interest income or withdrawn from savings. While no generic, industrial sector can enter stasis except while earning a standard return on its asset turnover, a household sector can persist with wages above consumption by the amount required to service a continuing level of indebtedness.

Figure 18-1 shows a generic industrial sector IK's capital account κ_{IK} \$ continuously relieved by debt service on the left and continuously charged on the right with financial services $\varepsilon_{IK} = \iota_K \cdot \kappa_{IK}$, where ι_K is minus the interest rate. An economy K's interest rate $-\iota_K$ operates in the negative to reflect the positive feedback attaching to financial positions: un-serviced debts and un-redeemed savings grow exponentially at the rate of interest.

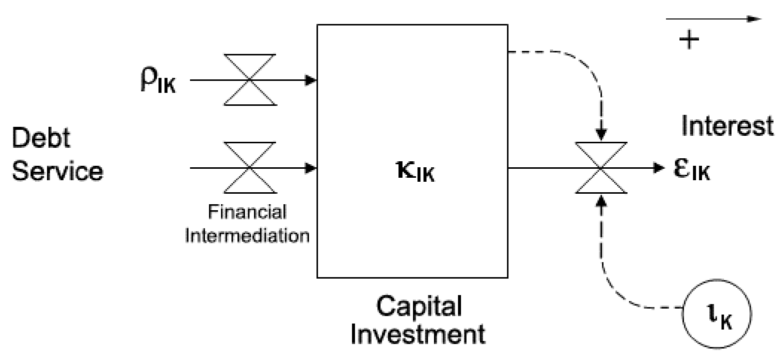


Figure 18-1: Control of a Capital Position

By our convention, capital obligations κ_{IK} are positive quanta; and (minus) financial services ε_{IK} (being formed by the product of a positive and a negative) are negative. The action of financial services ε_{IK} therefore reverses the directional arrow of Figure 18-1's sign convention, and operates to heap-up financial obligations κ_{IK}. (Minus) debt service ρ_{IK} must also compute to a negative, which again reverses the direction of the arrow illustrating its flow in Figure 18-1, and therefore operates to reduce financial obligations.

Inspection of Figure 18-1 points up the critical problem to be addressed in controlling the dynamics of economic adjustment:

- We know that R_{IJK} drives a physical process that is weakly self-reinforcing: though technical utility diminishes at the margin of an input's application, production nonetheless rises with rising inputs. Economic science is thereby obligated to explain why production should not increase without limit, even with constant technology.

- But any minimally faithful construction of economic control's financial component begins by compounding the problem. Financial positions are strongly self-reinforcing – which leaves us with a concept of economic dynamics as governed by the interaction of two positive feedback systems.

Debt service must control the level of financial obligations κ_{IK} toward the ultimate goal of equating earnings $-\rho_{IK}$ with dividends $-\varepsilon_{IK}$ at economy K's current interest rate $-\iota_K$, while also resolving itself to $\iota_K \cdot \kappa_{IK}$ just as financial stasis arrives. Debt service must therefore be decomposed into two terms: one of which is ρ_{IK} itself; and the other we shall call financial intermediation. To this point, all we know of financial intermediation is that it somehow stabilizes the financial adjustment system until it contrives to vanish at stasis. Elucidation of the intermediary cannot commence until Article 25 – after fully establishing SFEcon's method for price computation.

19. <u>Savings</u>

In all, an industrial sector's capital account κ_{IK} is the residual of what business have spent but not yet earned, i.e., the integrated difference between sales and costs, including the costs of capital. Definition of our financial state variables now turns to the control of households' savings γ_{LK}, the residual of what households have earned but not yet spent, i.e., the integrated difference between income (wages plus interest) and consumption.

One of neoclassicism's more stalwart positions opposite its heterodox antagonists maintains that savings must equal investment: that a dollar cannot be spent on credit until another dollar is diverted from immediate consumption into savings; hence all κ_{IK} must always be exactly offset by all γ_{LK}. Heterodoxy's position to the contrary has it that capitalism cannot be described other than by a persistent leveraging of investment beyond savings. Though SFEcon does not exist without the neoclassical premise of marginalism, we have no difficulty incorporating what we regard as superior holding of heterodoxy on the question of leverage into our normative macroeconomics.

Our deferred treatment of financial intermediation will establish that the idea of leverage has a dynamic counterpart in the notion of an investment term T years for saved funds. Figure 19-1 shows the financial state γ_{LK} of a household sector LK disaggregated into the elements of a third-order delay. As with our model asset of depletion (Figure 11-2) we instantiate a pipeline delay with three levels: A, B, and C – the sum of which defines savings γ_{LK}.

By convention, savings positions γ_{LK} are negative quanta. Their magnitude is further diminished by the action of a positive ρ_{LK}, i.e.: passive income – which is the difference between consumption and wages. Ideally, consumption can be greater than wages because it is partially offset by dividends ε_{LK}. Dividends augment savings by the action of positive ε_{LK}, which further decrements a negative γ_{LK}.

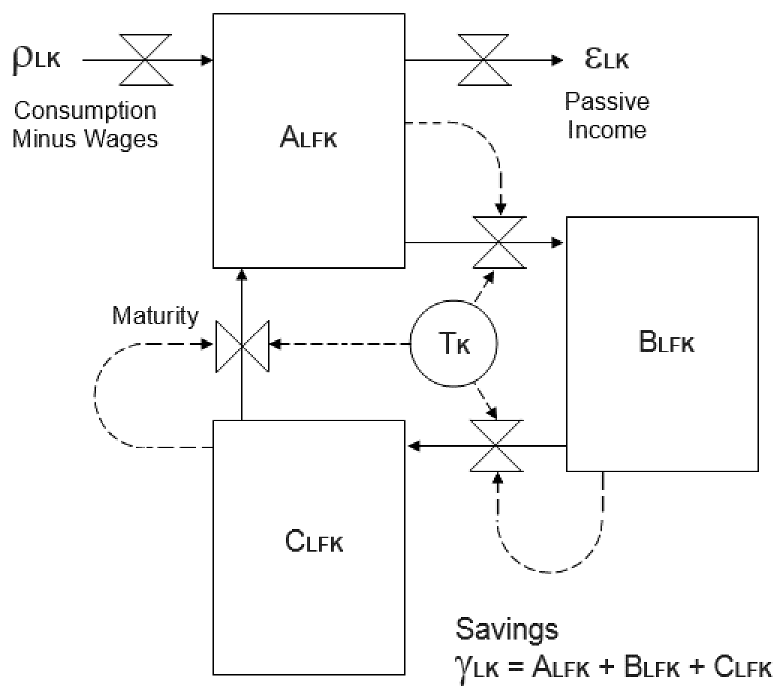


Figure 19-1: The Circuit of Loanable Funds

But savings, unlike physical assets, are not exhausted after their logistical term of $1/V_J$ years has run. Presumably one's principal is returned after a stated investment term, giving one the option to reinvest by adding-to or drawing-from that principal. Figure 19-1 models this process by returning the effluent from level C_{LFK} to level A_{LFK} through the action of a rate labeled **MATURITY**.

Level A_{LFK} is the point at which adjustments to savings can be injected into the loop. Interest payments ε_{LK} enter and are more or less offset by consumption-minus-wages ρ_{LK}, which controls savings levels γ_{LK} in analogy to the manner in which debt service controls investment levels κ_{IK}. At stasis, ε_{LK} and ρ_{LK} must equal one another, and the investment term T_K must be such that the effluent from A_{LFK} is exactly offset by inflow from C_{LFK}. This is to say that stasis requires savings be uniformly distributed around Figure 19-1's circuit of loanable funds.

20. The Investment Term

Articles 18 and 19 introduced Model 0's financial state variables. Industrial sectors' investment levels κ_{IK} and household sectors' savings levels γ_{LK} were shown to absorb the sectors' respective net cash flows $-\rho_{IK}$. Cash flows were defined by Equation 18-1. The generation of dividends $-\varepsilon_{IK}$ was portrayed in Figure 18-1. And the receipt of dividends $-\varepsilon_{LK}$ was portrayed in Figure 19-1.

Model 0 is limited to a single financial intermediary per economy K. One interest rate $-\iota_K$ controls all of K's capital investment positions κ_{IK}, and one investment term T_K controls all of K's savings positions γ_{LK}. K's intermediary is presumed to maintain contra-accounts κ_{0K} and γ_{0K} opposite these financial positions:

20-1) $$\kappa_{0K} = -\sum_{I=1}^{L-1} \kappa_{IK}$$

20-2) $$\gamma_{0K} = -\sum_{I=L}^{N} \gamma_{IK}$$

It will be noted that 1) κ_{0K} is a negative quantity; 2) γ_{0K} is a positive quantity; and 3) nothing requires that these quanta offset one another. Leverage, $-\kappa_{0K}/\gamma_{0K}>1$, is (as will be shown) the norm in capitalist systems: it is a requisite of capital development, and it persists in a stable equilibrium.

Figure 18-1's interest rate $-\iota_K$ and Figure 19-1's investment term T_K are brought together in the familiar notion of net present value **NPV**:

20-3) $$NPV = \frac{1}{(1-\iota)^T}$$

(Subscripts K and 0 are hereafter eliminated for notational convenience on the understanding that these variables belong to the intermediary of a given economy.)

Equation 20-3's transcendental expression of net present value is incomplete from the standpoint of our analysis because it carries no awareness of economy K's leverage, which is expressed in the intermediary's unequal investment κ and savings γ contra-accounts. These considerations are brought out in Equation 20-4's algebraic expression of net present value:

20-4) $$NPV = \frac{\gamma/T}{\gamma/T + \iota\cdot\kappa}$$

Having narrowed our view to a single intermediary managing a portfolio composed of all an economy K's productive assets α_K, we have conjured circumstances in which all returns on investment $-\iota\kappa$ accrue to savings of γ that are committed for T years. Equation 20-4 expresses **NPV** as a cash flow of $\gamma/T+\iota\kappa$ for delivery in T years that is continually being purchased for a cash flow of γ/T that is being tendered now. Risk is introduced to the model insofar as ι and T are variables, and the value of the currency in which savings γ are denominated is also subject to change during the investment term.

Equation 20-4 can be transformed as follows so as to isolate the interest rate $-\iota$, investment term T, and leverage $-\kappa/\gamma$:

20-5) $$NPV = \frac{1}{1+\iota\cdot T\cdot\kappa/\gamma}$$

Noting that 1) κ and γ are state variables, and 2) the interest rate $-\iota$ was determined in Equation 17-3, it is obvious that **NPV** can be eliminated from Equations 20-3 and 20-5 in order to calculate the investment term T. While closed-form calculations of T are not possible with this system, its practical application can be achieved through a Newton-Raphson approximation to its simultaneous solution.

The investment term T's definitions in Equations 20-3 and 20-5 offer some useful interpretations of a financial collapse. Solving these equations for T . . .

20-6) $$T = \frac{-\ln(\mathrm{NPV})}{\ln(1-\iota)}$$

20-7) $$T = \frac{\gamma}{(-\iota \cdot \kappa)}\left[\frac{\mathrm{NPV}-1}{\mathrm{NPV}}\right]$$

. . . reveals that a trivial simultaneous solution is always available at T=0 and **NPV**=1 irrespective of κ and γ's current levels, and suggests that the hazard of encountering this solution becomes a certainty as $-\iota$ approaches zero. An investment term of T=0 years constitutes a singularity analogous to catastrophic collapse of a capitalistic system by implosion of the savings loop portrayed in Figure 19-1.

This interpretation of capitalism's destructive singularity would have the savings loop of Figure 19-1 attempting to evacuate itself totally and instantly (T=0) yet remain completely inert (T=∞). Obviously the only physical counterpart to these abstractions is for savings γ to be null so as to allow this 'nothing' to move with infinite speed. A capitalist epoch presumably ends when every intermediary's interest rate has vanished. Savings positions might endure as bookkeeping entries under such circumstances, but their reality in terms of convertibility to objective commodities will have ended. As the realization of this fact dawns, the contradictions implicit in T's computation will be manifested.

Returning to consider the case of T and **NPV** operating at tractable values, we turn to the computations by which the industrial sectors' dividends $\varepsilon_{IK} = \iota_K \cdot \kappa_{IK}$ are distributed among the household sectors in proportion to their savings positions γ_{LK}. Solving Equation 20-5 for the intermediary's harvest of returns $\iota\kappa$ yields the return to his savings contra-account γ:

20-8) $$-\iota \cdot \kappa = \left[\frac{1 - 1/\mathrm{NPV}}{T}\right] \cdot \gamma$$

Obviously total returns $\iota\kappa$ can be scaled by substituting a household sector LK's portion γ_{LK} of total savings for Equation 20-8's $-\gamma$. This then computes a labor/leisure sector LK's passive income ε_{LK} as:

20-9) $$\varepsilon_{LK} = \left[\frac{1 - 1/NPV_K}{T_K} \right] \cdot \gamma_{LK}$$

Thus SFEcon's offering of a primitive instance of financial intermediation has an economy K's single intermediary continuously reckoning K's interest rate $-\iota_K$ in order to set K's investment term T_K so as to completely nullify his net returns:

20-10) $$\varepsilon_{0K} = -\iota_K \cdot \kappa_{0K} + \left[\frac{1 - 1/NPV_K}{T_K} \right] \cdot \gamma_{0K} = 0$$

VII. Price

21. General Price Levels

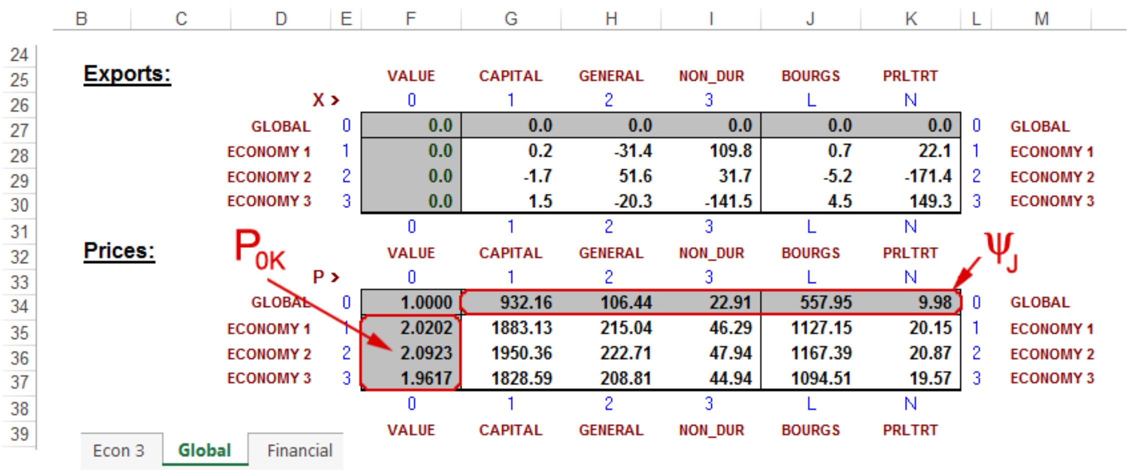
Model 0's physical state variables A_{IJK}, B_{IJK}, and C_{IJK} have now been defined in terms of the rates by which they are augmented and depleted. Those rates have been shown to reference money prices, and money has been arrayed in financial state variables κ_{IK} and γ_{LK} that depict investment and savings levels.

If prices are to be discovered functioning as a sort of pheromone capable of organizing the macroeconomic superorganism, then they would each express an economy K's entire current state, as well as all its parameters. Every price would therefore contain the same information, i.e.: all of it; and prices would differ from one another as to how that information is organized to express a given commodity J's relation to a given economy K.

Our neoclassical frame of reference requires that domestic prices P_{JK} must present themselves in the same proportions in every economy K for stasis to persist. This requisite can be visualized in Figure 21-1, which arrays the model's money prices P_{JK} with its export/import profiles X_{JK} at a point of stasis.

When in global stasis, any economy K's price vector should operate on any economy K's export profile to assure that no net export or import of value can be expressed in any currency:

21-1) $$0 = \sum_{J=1}^{N} \left(P_{JK} \cdot X_{JK} \right)$$



Exports:

X >		VALUE 0	CAPITAL 1	GENERAL 2	NON_DUR 3	BOURGS L	PRLTRT N		
GLOBAL	0	0.0	0.0	0.0	0.0	0.0	0.0	0	GLOBAL
ECONOMY 1	1	0.0	0.2	-31.4	109.8	0.7	22.1	1	ECONOMY 1
ECONOMY 2	2	0.0	-1.7	51.6	31.7	-5.2	-171.4	2	ECONOMY 2
ECONOMY 3	3	0.0	1.5	-20.3	-141.5	4.5	149.3	3	ECONOMY 3
		0	1	2	3	L	N		

Prices:

P >		VALUE 0	CAPITAL 1	GENERAL 2	NON_DUR 3	BOURGS L	PRLTRT N		
GLOBAL	0	1.0000	932.16	106.44	22.91	557.95	9.98	0	GLOBAL
ECONOMY 1	1	2.0202	1883.13	215.04	46.29	1127.15	20.15	1	ECONOMY 1
ECONOMY 2	2	2.0923	1950.36	222.71	47.94	1167.39	20.87	2	ECONOMY 2
ECONOMY 3	3	1.9617	1828.59	208.81	44.94	1094.51	19.57	3	ECONOMY 3
		0 VALUE	1 CAPITAL	2 GENERAL	3 NON_DUR	L BOURGS	N PRLTRT		

Figure 21-1: Prices and the Export/Import Profile

These considerations suggest that an economy K's money prices P_{JK} are the product of the global commodity values Ψ_J with P_{0K}, the current price of one value unit in the currency of economy K:

21-2) $$P_{JK} = P_{0K} \cdot \Psi_J$$

Having determined global commodity values Ψ_J in Article 16, the elucidation of prices is now reduced a determination of P_{0K}. Noting from Equation 16-3 that any and every commodity value Ψ_J embodies all of Model 0's physical state variables and parameters, we already have significant progress toward our goal of understanding each money price P_{JK} as an expression of the economic whole. It remains to formulate P_{0K} in terms of appropriate references the an economy K's entire financial state, as embodied by the monetary state variables κ_{IK} and γ_{LK}.

22. The Intersection of Prices with Utility

Computation of P_{0K} proceeds from a reconsideration of Equations 3-4. There are $\eta = N+1$ equations in this set, where N is the number of a sector's inputs.

3-4) $$\begin{aligned} \zeta &= \pi \cdot (Z - Y) \\ &= P_1 \cdot (U_1 + E_1) \\ &= P_2 \cdot (U_2 + E_2) \\ &\vdots \\ &= P_N \cdot (U_N + E_N) \end{aligned}$$

Adding all these equations results in Equation 22-1.

22-1) $$\eta\zeta = \pi Z + \sum_{J=1}^{N} P_J U_J - \pi Y + \sum_{J=1}^{N} P_J E_J$$

This allows consolidation of the interaction between prices and a sector's utility parameters in a single variable υ:

22-2) $$\upsilon = \pi Z + \sum_{J=1}^{N} P_J U_J$$

and isolates an expression for the sector's dividend $-\varepsilon$ (in the negative: profits are coming 'back to us' out of economic activity):

22-3) $$\varepsilon = -\pi Y + \sum_{J=1}^{N} P_J E_J$$

Variables υ and ε then epitomize a sector's ζ as follows:

22-4) $$\zeta = \frac{\upsilon + \varepsilon}{\eta}$$

Adding-up the ζ's for all of an economy K's sectors I yields:

22-4) $$\sum_{I=1}^{N} \left(\eta_{IK} \cdot \zeta_{IK}\right) = \sum_{I=1}^{N} \upsilon_{IK} - \sum_{I=1}^{N} \varepsilon_{IK}$$

Equation 22-4 is simplified by elimination its last term because, as shown in Equation 20-10, the financial intermediary will presumably have chosen an investment term T_K such that all the dividends generated by industry will be exactly offset by the dividends awarded to households. We also note that Equations 22-2's υ_{IK} \$/yr. has a value equivalent in Υ_{IK} ~~G~~/yr. . . .

22-5) $$\Upsilon = \Psi_I Z_I + \sum_{J=1}^{N} \Psi_J U_J$$

. . . that can be used to bring P_{0K} into the analysis:

22-6) $$\upsilon_{IK} = P_{0K} \cdot \Upsilon_{IK}$$

Eliminating υ_{IK} and ε_{IK} from equations 22-4 and 22-6 yields an expression . . .

22-7) $$\sum_{I=1}^{N} \left(\eta_{IK} \cdot \zeta_{IK} \right) = P_{0K} \cdot \sum_{I=1}^{N} \Upsilon_{IK}$$

. . . that can be rearranged to isolate P_{0K}:

22-8) $$P_{0K} = \frac{\sum_{I=1}^{N} \left(\eta_{IK} \cdot \zeta_{IK} \right)}{\sum_{I=1}^{N} \Upsilon_{IK}}$$

Here the specification of P_{0K} lacks only an approach to ζ_{IK} that does not reference prices (which cannot be known until P_{0K} is known) but that derives from the sectors IK's financial states.

23. Value's Money Price

The approach to ζ from the standpoint dividends $-\varepsilon$ will be called β. References to ε begin with the equation of dividends to sales πY minus budgets σ:

23-1) $$-\varepsilon = \pi Y - \sigma$$

Recurring once again to Equations 3-4, first equality supplies the product's price π in the above by solving for π and substituting β for ζ:

23-2) $$\pi = \frac{\beta}{Z - Y}$$

Multiplying both sides of Equation 23-2 by Y supplies the middle term of Equation 23-1:

23-3)

$$\pi Y = \frac{\beta Y}{Z - Y}$$

and substituting β for the θ in Equation 15-3 supplies Equation 23-1's budget term σ:

23-4)

$$-\varepsilon = \frac{\beta Y}{Z - Y} - \beta \sum_{J=1}^{N} \frac{E_J}{U_J + E_J}$$

A bit of re-arranging then produces our desired expression for β:

23-5)

$$\beta_{IK} = \varepsilon_{IK} \Bigg/ \left(\frac{-Y_{IK}}{Z_{IK} - Y_{IK}} + \sum_{J=1}^{N} \frac{E_{IJK}}{U_{IJK} + E_{IJK}} \right)$$

It must be noted that a slightly different expression of β is required for a household sector. This is because a household's product –Y is leisure time, which (per Article 10) must be subtracted from the household's total experience of time τ in order to arrive at remunerated labor. Household's equivalent to Equation 23-1 is therefore:

23-6)

$$-\varepsilon = \pi(\tau + Y) - \sigma$$

Equation 23-2 also changes to reflect that the marginal cost of producing leisure is **minus** the wage π:

23-7)

$$\pi = -\frac{\beta}{Z - Y}$$

Retracing the algebra of Equations 23-2 to 23-5 leads to the following expression for a household's β:

23-8)

$$\beta_{LK} = \varepsilon_{LK} \Bigg/ \left(\frac{\tau_{LK} + Y_{LK}}{Z_{LK} - Y_{LK}} + \sum_{J=1}^{N} \frac{E_{LJK}}{U_{LJK} + E_{LJK}} \right)$$

Values of β_{IK} can now be substituted for those of ζ_{IK} in Equation 22-8 to specify value's price P_{0K} in economy K . . .

23-9)

$$P_{0K} = \frac{\sum_{I=1}^{N} \left(\eta_{IK} \cdot \beta_{IK} \right)}{\sum_{I=1}^{N} \Upsilon_{IK}}$$

. . . which enables computation of cardinal prices P_{JK} via equation 21-2:

21-2)

$$P_{JK} = P_{0K} \cdot \Psi_J$$

P_{0K} having been established, all the of the financial parameters describing a given economy K – currency value, interest rate, investment term, and leverage – are now known throughout time. These can be inspected, along with the monetary levels and rates they control, through the Excel emulator's FINANCIAL portal:

Economy #1:

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
12				CAPITAL			DIVIDENDS			NET CASH		MATURING		SAVINGS	
13	P_{01} >	2.0202		ACCOUNTS		GENERATED		RECEIVED		FLOWS		INVESTMENTS		BALANCES	
14				-452126	0	45203	0	-45203	0	0	0	0	0	338013	
15	$-\iota$ >	10.00%	CAPITAL	171556	1	-17152	1	0	1	17152	1	0	1	0	CAPITAL
16			GENERAL	143966	2	-14393	2	0	2	14393	2	0	2	0	GENERAL
17	Term >	6.7522	NON_DUR	136603	3	-13657	3	0	3	13657	3	0	3	0	NON_DUR
18			BOURGS	0	L	0	L	40548	L	-40547	L	-44905	L	-303206	BOURGS
19	$-\kappa/\gamma$ >	1.3539	PRLTRT	0	N	0	N	4655	N	-4655	N	-5155	N	-34807	PRLTRT
20	leverage			κ, \$		ε (–), \$/yr		ε (+), \$/yr		$-\rho$, \$/yr		γ/T, \$/yr		γ, \$	
21															

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Figure 23.1

24. The Demand for Value

Article 14 introduced the notion of an artificial value unit that exists in a fixed quantity and is propagated among the sectors IK in column J=0. E_{I0K} was defined as expressing the current value of assets being turned-over by sector IK:

14-1)

$$E_{I0K} = -\sum_{J=1}^{N} \left(\Psi_J \cdot E_{IJK} \right)$$

Article 16 derived absolute commodity values Ψ_J on the presumption that $E_{I0K}=V_0\cdot(A_{I0K}+B_{I0K}+C_{I0K})$ was known to the analysis. With P_{0K} known from Article 23, the method by which sectors IK might be properly evaluated can be set forth. We proceed on the model of Section IV, where control Model 0's physical rates and levels were described by the signal path of Figure 11-3.

Here we see replenishment rates R_{IJK} driving the circuit by which goods are produced and taken into inventories to more or less replenish that which is extinguished by production. Article 12 established the current optimal rate Q_{IJK} for IK to use J:

12-2) $$Q_{IJK} = \frac{\zeta_{IK}}{P_{JK}} - U_{IJK}$$

And Article 13 defined R_{IJK} as equaling Q_{IJK} whenever the availability of J is sufficient, and as rationing the availability of J according to the proportions of Q_{IJK} when J is insufficient.

To be operative from the standpoint evaluating the sectors IK, Q_{I0K} should receive a negative sum on the value equivalents of the demands Q_{IJK} being expressed in IK's row:

24-1) $$Q_{I0K} = -\sum_{J=1}^{N}\left[\Psi_{JK}\left(\frac{\zeta_{IK}}{P_{JK}} - U_{IJK}\right)\right]$$

Equation 24-2 recasts Equation 12-2's definition of Q_{IJK} for this special case of J=0:

24-2) $$Q_{I0K} = \frac{-N_{IK} \cdot \zeta_{IK}}{P_{0K}} - U_{I0K}$$

Here we see that the number of ζ_{IK}'s in row IK is given by N_{IK}; Ψ_J/P_{JK} has been replaced by $1/P_{0K}$ in every term of the sum; and U_{I0K} is defined per Equation 24-3:

24-3) $$U_{I0K} = -\sum_{J=1}^{N}\Psi_J \cdot U_{IJK}$$

Specification of Q_{I0K} immediately defines R_{I0K}, which (in exception to R_{IJK}, which either equals Q_{IJK} or is rationed) must continuously redistribute **all** of value's constant flow Y_{00} among the sectors IK according to the **proportions** among the Q_{I0K}'s. Thus the sum of E_{I0K} over all IK will always equal $-Y_{00}$.

25. Financial Intermediation

The essentially neoclassical model set out to this point will not exhibit dynamic stability. It can continue a perfectly poised initial steady-state for centuries; it can faithfully reproduce responses to stimuli for decades; but it will ultimately dissolve into chaos. These deficiencies are instructive insofar as their correction requires some useful inquiries into the natures of money and financial control.

In addition to discerning an economy's interest rate $-\iota_K$ and calculating its investment term T_K, the financial intermediary must move funds among the industrial sectors' capital positions κ_{IK} so as to guide an economy toward some unifying end-state. Figure 25-1 presents an instance of the hypothetical end-state envisioned by classical economists, viz.: each commodity J's price equals its marginal cost of production, as well as its value of marginal product among all the sectors that use it.

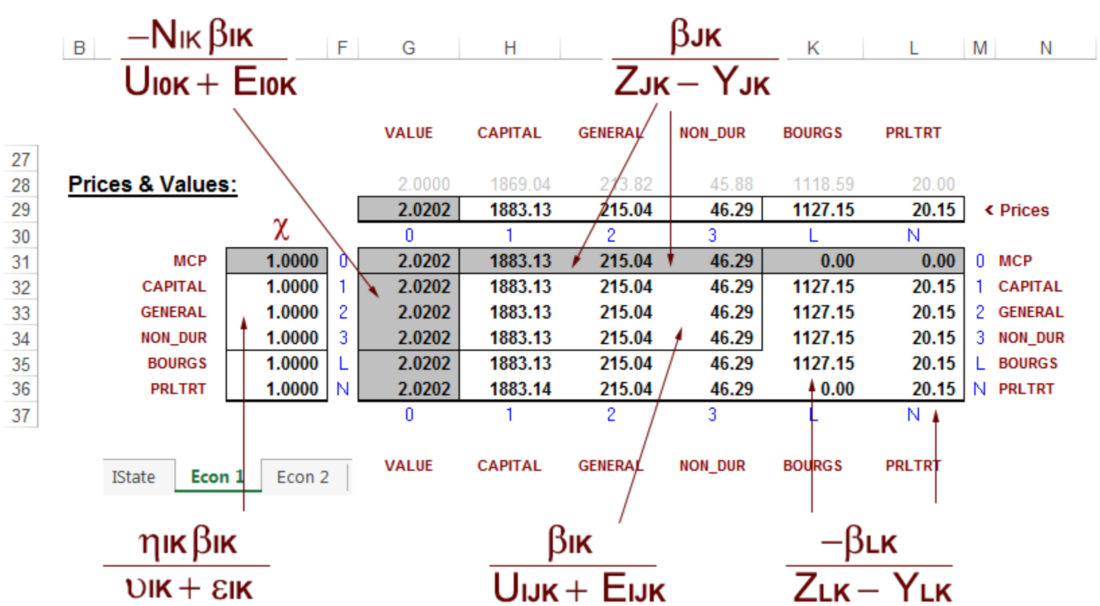

Figure 25-1: Prices, Costs, and Values

Commodity J's values of marginal product among the sectors IK, $\beta_{IK}/(U_{IJK}+E_{IJK})$, appear in the body this matrix. Industrial sector's marginal costs of production, $\beta_{JK}/(Z_{JK}-Y_{JK})$, are shown in row 0; and household sectors L's marginal costs of production, $-\beta_{LK}/(Z_{JK}-Y_{JK})$, are shown on the diagonal. Marginal values of value are computed in column 0. Ultimate control of Model 0 will rely on the sectors' marginal values of money χ_{IK} in the vertical axillary vector:

25-1) $$\chi_{IK} = \frac{\eta_{IK} \cdot \beta_{IK}}{\upsilon_{IK} + \varepsilon_{IK}}$$

For industrial sectors η_{IK} is $N_{IK}+1$, the number of IK's inputs N_{IK} plus 1. This count includes a β_{IK} for each computation of IK's values of marginal product, plus one more β_{IK} to include computation of its marginal cost of production. Note that a household sector's β_{LK} enters the marginal cost of production computation as a negative, so households' η_{LK} is $N_{LK}-1$.

An industrial sector having a greater marginal value of money than another is the better investment; so financial intermediation that adjusts capital positions κ_{IK} in search of maximal returns should operate to equalize the χ_{IK}'s among the sectors – which can only happen when all the sectors' marginal values of money align with money's spot price of unity. Model 0's stated objective being to establish the simplest operative representation capitalism's dynamics suggests Figure 25-2's matrix formulation of the data needed for successful financial intermediation.

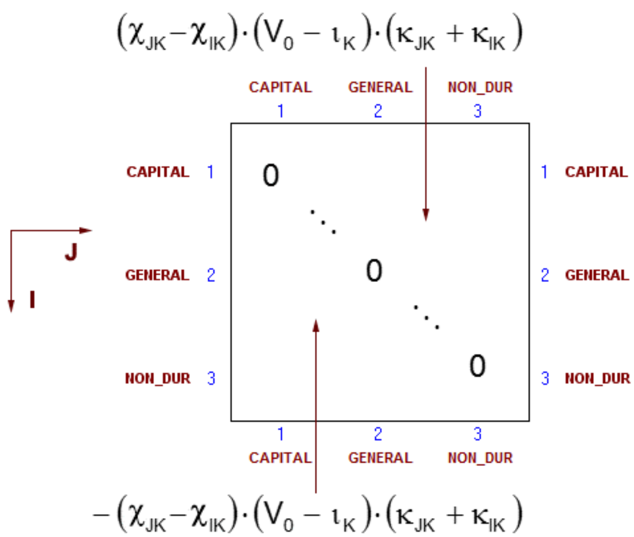

Figure 25-2:
Matrix of Financial Intermediation

If money is to be usefully transferred between two industrial sectors IK and JK, the rate of transfer would logically be proportional to the sum of these sector's capital positions $\kappa_{JK}+\kappa_{IK}$. This quantity would be propelled by the quantity $V_0-\iota_K$, which transforms a capital position into a sector's rate of expenditure for asset replenishment plus its expected profit. The third factor going into each element of this matrix would be the difference between their marginal values of money $\chi_{JK}-\chi_{IK}$.

Diagonal elements in the intermediation matrix are of course null because flows from a capital position to itself would be meaningless. In an instance where a cell in the upper triangle of this matrix expresses a flow into a sector IK from a sector JK, that cell's reciprocal in the lower triangle receives its negated value in order to express the flow out of JK. Thus the sum of all flows in the intermediation matrix are always null. While the intermediary does create and annihilate capital by offsetting entries to his counter-account, every credit must be offset by a debit. There can be no ‘one-sided’ journal entries: an intermediary is not allowed to move money from nowhere to somewhere.

A sum across the entries for a given row IK of Figure 25-2’s ‘FI matrix’ computes the net of all the intermediary’s additions and subtractions for a given capital position κ_{IK}. This sum can now be used to complete Figure 18-1’s depiction of the elements by which an industrial sector’s capital position is controlled:

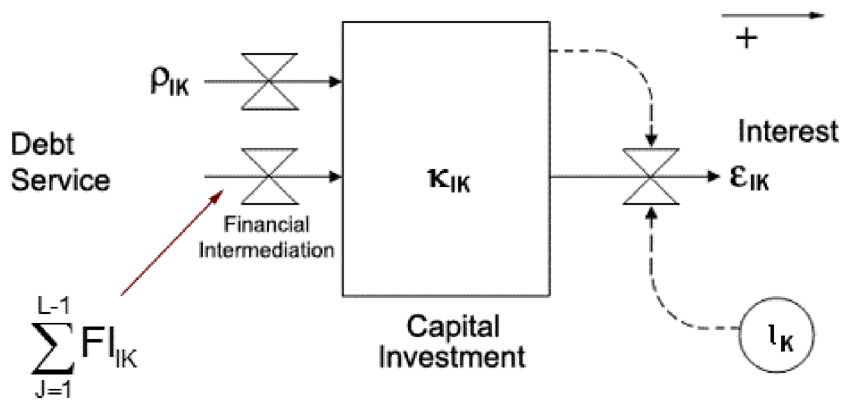

Figure 25-3: Financial Intermediation’s Control of an Investment Position

Noting that Equation 25-1 equates the financial discriminants β_{IK} and $(\upsilon_{IK}+\varepsilon_{IK})/\eta_{IK}$ when χ_{IK} is unity, we can complete all of Model 0’s approaches to the financial discriminant with Figure 25-4’s epitome of the FINANCIAL portal:

	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF
12								υ + ε					
13		β		θ		ζ		η		Θ		Θ	
14			0		0		0	0	0		0	0	
15	CAPITAL	1146791	1	1146791	1	1146791	1	1146791	1	569143	1	567669	CAPITAL
16	GENERAL	976324	2	976324	2	976324	2	976324	2	484689	2	483287	GENERAL
17	NON_DUR	885574	3	885574	3	885574	3	885574	3	439415	3	438365	NON_DUR
18	BOURGS	71271	L	71270	L	71271	L	71271	L	35373	L	35279	BOURGS
19	PRLTRT	39496	N	39496	N	39496	N	39496	N	19599	N	19551	PRLTRT
20		$/year		$/year		$/year		$/year		G/year		G/year	
21													

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Figure 25-4: Convergence of the Financial Discriminants

26. Issues in Household Utility

Article 10 placed the household sectors within SFEcon's matrix structure, but the complete details of households' accommodation to classical causality could not be fully set forth until substantially all else was in place.

Section IV postulated an economic dynamics sourced in the replenishment of assets being expended in current production. It is, however, in the nature of a labor/leisure sector's product that it cannot be held in inventory. A moment perishes at the instant of its creation. If a person who is willing and able to work cannot find a market for his services, then the time in which he might have been productive is lost forever.

Industrial sectors' 'stocks' of the household output must therefore bear interpretation as something like a habituation among a segment of the population to report to work at a given place, their training for the work to be done there, and their being domiciled near their place of work. All together these assets tend to be reported as 'goodwill' or the value that an enterprise has as a growing concern. The labor/leisure 'commodity's' turnover fraction V_L would then be interpreted as controlling the delay associated with attracting workers to a different employer, a different locale, or a different profession.

Special consideration must also be given to a household sector's intake of its own product. At stasis, R_{LLK} receives a labor/leisure sector's production of leisure $-Y_{LK}$. When sector LK's labor is in short supply then labor's availability $\tau_{LK}+Y_{LK}$ is rationed among the other sectors demanding it, and R_{LLK} is once again $-Y_{LK}$. But when the demand for labor falls short of $\tau_{LK}+Y_{LK}$, some amount of leisure in excess of $-Y_{LK}$ is involuntary and must be forced into R_{LLK}. This accomplished by redefining the household R_{LLK} of Equation 13-8 as simply the residual of every R_{ILK} in column L (including X_{LK}) with the $-\tau_{LK}$ at R_{0LK}.

A more lengthy consideration is necessitated by a minor eccentricity in the neoclassical view. While many informative SFEcon models have been created with household utility described along the lines set forth to this point, none of them have fully satisfied the expectations of neoclassical theory, i.e.: the presumption that, for any properly conditioned set of utility parameters, there should exist at least one corresponding set of prices and physical exchanges vindicating the premises of general optimality. The experience upon which we challenge this view requires a bit of introduction.

SFEcon demonstrations are typically begun at a general economic optimum; are driven into chaotic behavior by some combination of exterior stimuli; and are then observed to re-establish generally optimal physical exchanges and prices. Stimuli can be either elastic or plastic.

- Elastic stimuli exogenously change a state variable such as the quantity of a good owned by a sector or the quantity of a good on the market. The model responds to these stimuli by recreating the exact physical state that characterized the initial optimum, with the same proportions among commodity prices, but with prices occurring at a different general level.

- Plastic stimuli change one or more utility parameters. These impulses require that a model respond by discovering an entirely new physical state, with new proportions among commodity prices that nonetheless fulfill all the criteria of a different, but no less unique and stable, optimum.

As defined thus far, primitive versions of Model 0 respond to elastic stimuli exactly as neoclassicism requires. But, when responding to plastic stimuli such as an isolated improvement in manufacturing technique, these models' emulations never quite settle into a perfect optimum. Rather, they persist in a new physical stasis characterized by: 1) a small but fixed portion of the time continuum claimed by neither labor nor leisure, with 2) fixed amounts of industrial products on the market that 3) somehow fail to induce the temporary surge of demand over supply that might relieve the imbalance.

Such experiments are nonetheless promising in that they portray phenomena of much concern to economists. Article 10's formalization of household macroeconomics offers a number of interpretations for these phenomena – the most obvious being that Figure 10-1 implies a downward-sloping supply-of-labor schedule. Further explanation shows that households' 'output' of time, being an exogenous constant τ, is not subject to economic calculation or adjustment. And we also observe that imposition of the parameter τ deducts one degree of mathematical freedom from that available to a generic industrial sector.

While instructive, the primitive model described to this point is overly restrictive in its portrayal of wages as rigid, rather than merely 'sticky', in those circumstances where money wages need to fall so that improvements in the economy's technical potential can become fully realized in a new general optimum. The model must therefore be advanced to embody a certain plasticity of households' behavior whereby their utility tradeoffs are modified to equate their wage with their leisure's marginal value.

Model 0's elementary expression of this phenomenon makes what would normally be the 'parameter' Z_L at U_{LL} into a variable. A variable Z_L will provide the degree of freedom lost when the output variable at R_{0LK} becomes the constant $-\tau_{LK}$, which then allows households to participate in a perfect general economic optimum.

It should be self-evident that a household will adjust the relation between its appetite for consumption and its desire for leisure until it is satisfied with its passive income from investments (or, as the case may be, with its level of indebtedness). Working more and consuming less in order to save more or reduce debt instantiates a household's recasting of the utility function by which it is made visible to economic science. SFEcon's monetary variables suffice for an elementary expression of this truism.

27. Households' Utility Variable

A household sector's withdrawal ρ_{LK} of passive income from its savings position does not operate on the same logic as debt service. Households are composed of free agents whose net cash flows are not subject to intermediation for the sake of optimal returns. Wages plus passive income minus consumption, **is** the definition of net additions to savings. Household optimality can, therefore, only be achieved by altering a household sector's relationship between its desire for leisure and its desire for consumption.

These adjustments **of** a household sector's utility function are somewhat analogous to a financial intermediary's adjustments **to** the shape of an economy's industrial production functions: both operations can be portrayed as responses to the financial signal of money's marginal value, i.e.: a separation of Equation 25-1's χ_{IK} from unity.

When a labor/leisure sector's χ_{LK} is unity, Equation 25-1 can be transformed an equality between two approaches to the financial discriminant:

27-1) $$\beta_{LK} = \frac{\upsilon_{LK} + \varepsilon_{LK}'}{\eta_{LK}}$$

Equation 27-1 can be rearranged to compute a labor/leisure sector LK's optimal current dividend ε_{LK}':

27-2) $$\varepsilon_{LK}' = \eta_{LK} \cdot \beta_{LK} - \upsilon_{LK}$$

Continuing with an instance of χ_{LK} equal to unity, a household sector's current β_{LK} would derive from a restatement of Equation 23-8 . . .

27-3) $$\beta_{LK} = \varepsilon_{LK}' / \omega_{LK}$$

. . . into which the variable ω_{LK} has been introduced for our notational convenience:

27-4) $$\omega_{LK} = \left(\frac{\tau_{LK} + Y_{LK}}{Z_{LK} - Y_{LK}} + \sum_{J=1}^{N} \frac{E_{LJK}}{U_{LJK} + E_{LJK}} \right)$$

Eliminating β_{LK} From Equations 27-2 and 27-3 determines ε_{LK}', a sector LK's optimal dividend given current prices, LK's current utility tradeoffs, and LKs current state:

27-5)
$$\varepsilon_{LK}' = \frac{\upsilon_{LK}}{\left(\eta_{LK}/\omega_{LK} - 1\right)}$$

Any separation between Equation 27-5's optimal passive income ε_{LK}' and the current passive income ε_{LK} of Figure 19-1 is to be resolved by re-shaping household utility.

It must be noted that economics proper has long regarded household utility as having no unit of measure, hence no price, and therefore no possibility of quantification. In the alternative, we advance the variable Z_{LK} of Figure 10-1 as a cardinal measure of household utility. As shown in Figure 25-1, changing Z_{LK} alters the 'marginal cost' of producing labor; but its reformation of LK's utility function occurs without altering the shape of the utility isoquants governing tradeoffs among a sector LK's inputs (i.e.: items of household consumption). And Z_{LK} does not enter the market's valuation of labor via Equation 16-3.

Model 0 uses the difference between ε_{LK}' and ε_{LK} to create its primitive adjustment regime for Z_{LK}. This difference must first be scaled in respect to the price of LK's product, the wage rate P_{LK}. The term $(\varepsilon_{LK}' - \varepsilon_{LK})/P_{LK}$ has the units of labor, $\tau_{LK} + Y_{LK}$. A relationship between Z_{LK} and leisure $-Y_{LK}$ is then derived from Equation 3-1's generic hyperbolic production function:

27-6)
$$\left[\frac{Y}{Z}\right] = 1 - \prod_{\substack{J=1 \\ J \neq L}}^{N} \left(\frac{U_J}{(U_J + E_J)}\right)$$

And our further notational convenience will abbreviate this expression's last term as simply Π_{LK}. As shown in Figure 27-1, these specifications determine the driving function for a state variable representing a labor/leisure sector LK's Z_{LK}:

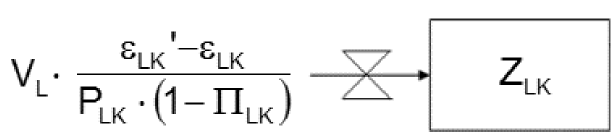
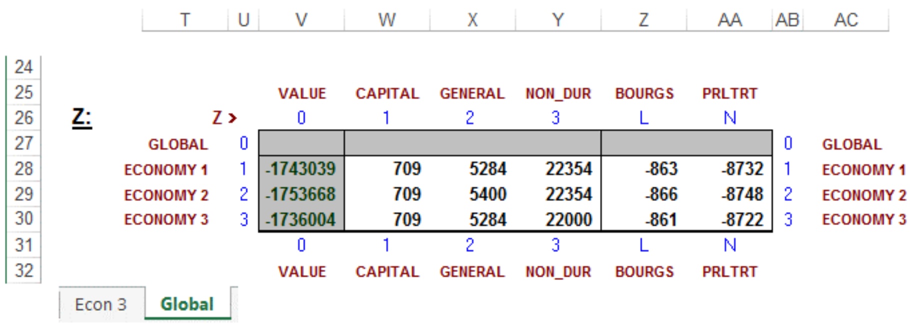


Figure 27-1: A Household Sector's Z_{LK} as a State Variable

If Figure 27-1's household Z is operative as a quantification of household utility, then utility's unit of measure is the labor hour; its price is (minus) the wage; and its manifestation is respite from work in the form of leisure.

As shown in Figure 27-2, the demonstration model's Z_{JK}'s are presented at the bottom right of the GLOBAL worksheet. For generic industrial sectors, the Z_{JK}'s are merely constants recapitulated from row zero of the utility matrix U_{IJK}. Households' Z_{LK}'s are creatures of the calculations described in this article.



Z:

	Z >	VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT		
		0	1	2	3	L	N		
GLOBAL	0							0	GLOBAL
ECONOMY 1	1	-1743039	709	5284	22354	-863	-8732	1	ECONOMY 1
ECONOMY 2	2	-1753668	709	5400	22354	-866	-8748	2	ECONOMY 2
ECONOMY 3	3	-1736004	709	5284	22000	-861	-8722	3	ECONOMY 3
		0	1	2	3	L	N		
		VALUE	CAPITAL	GENERAL	NON_DUR	BOURGS	PRLTRT		

Econ 3 | Global

Figure 27-2

Note that Z_{22} and Z_{33} present the parametric changes used to initiate the standard demonstration's dynamics: Z_{22} higher than other Z_{2K}'s means that economy K=2 is the superior producer of commodity J=2; and Z_{33} lower than the other Z_{3K}'s means that economy K=3 is the inferior producer of commodity J=3. Households' variable Z_{LK}'s present the ultimate effect that these changes in manufacturing technique have on households.

Appendices

A: General Equilibrium Prices

Our demonstration workbook operates on one and only one input/output structure. The model always has five sectors, three of which are industries, and two of which are households. Three input/output matrices describe the three national economies composing this model's theoretical world. The model will always be initialized with identical numerical content in each of the three matrices.

Specific numerical contents for the demonstration workbook are generated on the GPE worksheet. Experimenters are free to enter any information shown with a red typeface in

Figure A-1. This information includes the interior elements of the physical I/O matrix, an initial specification of the interest rate, the proletarian wage, and the price of one value unit.

'Investment Fractions' must be specified in order to make an initial division of interests in economy's asset base among the household sectors. Since there are just two household sectors in this model, only the bourgeoisie's fraction need be specified: proletarians automatically receive the remainder.

These specifications are sufficient to determine relative prices for all the model's commodities. Absolute prices can only be computed upon specification of one cardinal price with which to scale the others. As shown in Figure A-1, the proletarian wage P_N is set by the experimenter. The general price level P_0 does not, for purposes of model initiation, have anything to do with the magnitude of absolute prices. In setting P_0 the experimenter is merely specifying the initial number of absolute value units against which all else in the model will be measured.

General Price Equilibrium

	ρ	R	VALUE 0	CAPITAL 1	GENERAL 2	NON_DUR 3	BOURGS L	PRLTRT N		
+ XPT; - MPT			0	0	0	0	0	0		
VALUE	0	0	309552	-100	-750	-3200	-876	-8766	0	VALUE
CAPITAL	-16991	1	-84956	30	200	800	20	600	1	CAPITAL
GENERAL	-14578	2	-72892	25	100	700	30	600	2	GENERAL
NON_DUR	-13348	3	-66739	20	150	500	26	600	3	NON_DUR
BOURGS	40426	L	-62719	20	200	900	800	200	L	BOURGS
PRLTRT	4492	N	-22246	5	100	300	0	6766	N	PRLTRT
			0	1	2	3	L	N		

Interest rate		VALUE 0	CAPITAL 1	GENERAL 2	NON_DUR 3	BOURGS L	PRLTRT N	
10.00%	P	2.0000	1869.04	213.82	45.88	1118.59	20.00	Prices

Investment Fractions	
0.9000	0.1000
L	N

Compute Prices

Figure A-1: General Equilibrium Prices

Having provided all of Figure A-1's data that is set in red type (and only that data) the experimenter must click on the COMPUTE PRICES button in order to specify Prices 1 through L. Whenever any of these data are changed, new equilibrium prices must be computed before proceeding further in the initialization process. Failure to do so will likely result in an incoherent initial modeling state.

The algorithm invoked by clicking on COMPUTE PRICES is an obvious variation on a procedure already familiar in economics. As shown in Figure A-2, sectors' profits are formulated by multiplying a prospective price vector times an elementary transformation on the rates matrix in which a sectors' output has been subtracted from its diagonal element.

$$\begin{array}{c} 1\\ 2\\ 3\\ L\\ N \end{array} \begin{pmatrix} R_{11}-Y_1 & R_{12} & R_{13} & R_{1L} & R_{1N} \\ R_{21} & R_{22}-Y_2 & R_{23} & R_{2L} & R_{2N} \\ R_{31} & R_{32} & R_{33}-Y_3 & R_{3L} & R_{3N} \\ R_{L1} & R_{L2} & R_{L3} & R_{LL}-\tau_L & R_{LN} \\ R_{N1} & R_{N2} & R_{N3} & R_{NL} & R_{NN}-\tau_N \end{pmatrix} \times \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_L \\ P_N \end{pmatrix} =$$

$$-\iota \times \begin{pmatrix} \left\{ -P_1R_{11} - P_2R_{12} - P_3R_{13} - P_LR_{1L} - P_NR_{1N} \right\} \\ \left\{ -P_1R_{11} - P_2R_{12} - P_3R_{13} - P_LR_{1L} - P_NR_{1N} \right\} \\ \left\{ -P_1R_{11} - P_2R_{12} - P_3R_{13} - P_LR_{1L} - P_NR_{1N} \right\} \\ \left\{ +P_1\begin{pmatrix} R_{11}+ \\ R_{21}+ \\ R_{31} \end{pmatrix} +P_2\begin{pmatrix} R_{12}+ \\ R_{22}+ \\ R_{32} \end{pmatrix} +P_3\begin{pmatrix} R_{13}+ \\ R_{23}+ \\ R_{33} \end{pmatrix} +P_L\begin{pmatrix} R_{1L}+ \\ R_{2L}+ \\ R_{3L} \end{pmatrix} +P_N\begin{pmatrix} R_{1N}+ \\ R_{2N}+ \\ R_{3N} \end{pmatrix} \right\} \times C_L \\ \left\{ +P_1\begin{pmatrix} R_{11}+ \\ R_{21}+ \\ R_{31} \end{pmatrix} +P_2\begin{pmatrix} R_{12}+ \\ R_{22}+ \\ R_{32} \end{pmatrix} +P_3\begin{pmatrix} R_{13}+ \\ R_{23}+ \\ R_{33} \end{pmatrix} +P_L\begin{pmatrix} R_{1L}+ \\ R_{2L}+ \\ R_{3L} \end{pmatrix} +P_N\begin{pmatrix} R_{1N}+ \\ R_{2N}+ \\ R_{3N} \end{pmatrix} \right\} \times C_N \end{pmatrix} \begin{array}{c} 1\\ 2\\ 3\\ L\\ N \end{array}$$

Figure A-2: The Price Computation Mechanism

This formulation is then equated to the profits that the sectors must earn at equilibrium. Industries must earn a return on asset flows equal to the interest rate $-\iota$. Households receive these returns in proportion to the coefficients C_L and C_N describing the sectors' respective investment fractions for the model's initial state. The system is solved by isolating the price vector, arbitrarily setting the proletarian wage P_N to make the system determinant, normalizing the ensuing matrix, and extracting the remaining prices by Gaussian elimination.

B: State Variables

The ISTATE worksheet requires minimal interaction from the experimenter because physical rates of change and the rate of interest are automatically brought forward from the GPE sheet. As shown in Figure B-1, an experimenter interacts with these data by specifying turnover fractions V_J/yr as defined in Article 11. Specification of the model's time constants includes the investment term T yrs., as defined in Article 20.

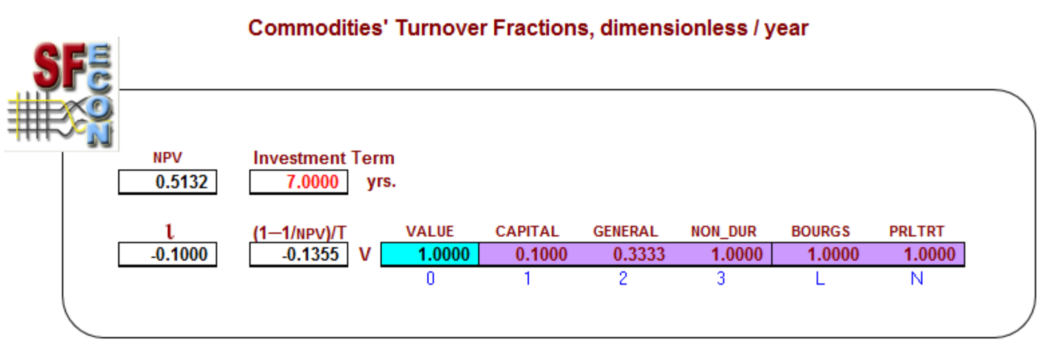


Figure B-1: Model 0's Turnover Fractions

Model 0 operates with the exterior specification of the V_J's on the ISTATE worksheet. V_0 is always equal to unity: this is necessary to the model's dimensional cohesion, uniting the per annum unit of the interest rate $-\iota$ and the measure of the investment term T in years. The remaining elements of V_J, those shown in red, may be set so as to impart to any desired logistical identity to the Commodities J.

The ISTATE worksheet generates contents for Figure 11-2's higher-ordered delay mechanism at each cell of the matrix structure, as elaborated in Figure B-2. Note that determinations of money prices, the interest rate, and the investment term amount to a complete specification of the system's financial state. Initial values for the savings levels γ (distributed in Figure 19-1's savings pipelines) are shown in a vertical auxiliary vector. Another vertical auxiliary vector receives initial values for the industrial sectors' money capital κ, (as shown in Figure 18-1). The system's state is completed by inclusion of Z_L's for the two labor/leisure sectors at the bottom right.

When the model is reset, or when it completes its emulation of a time-span, it writes out each economy K's current state on its respective worksheet ECON 1, ECON 2, or ECON 3. Whenever the SIMULATE button is clicked, the economy's state is read back into the emulation program to provide an initial state for the next interval to be simulated. This facility is intended allow exogenous elastic stimuli to be written into the worksheets for any of the economies.

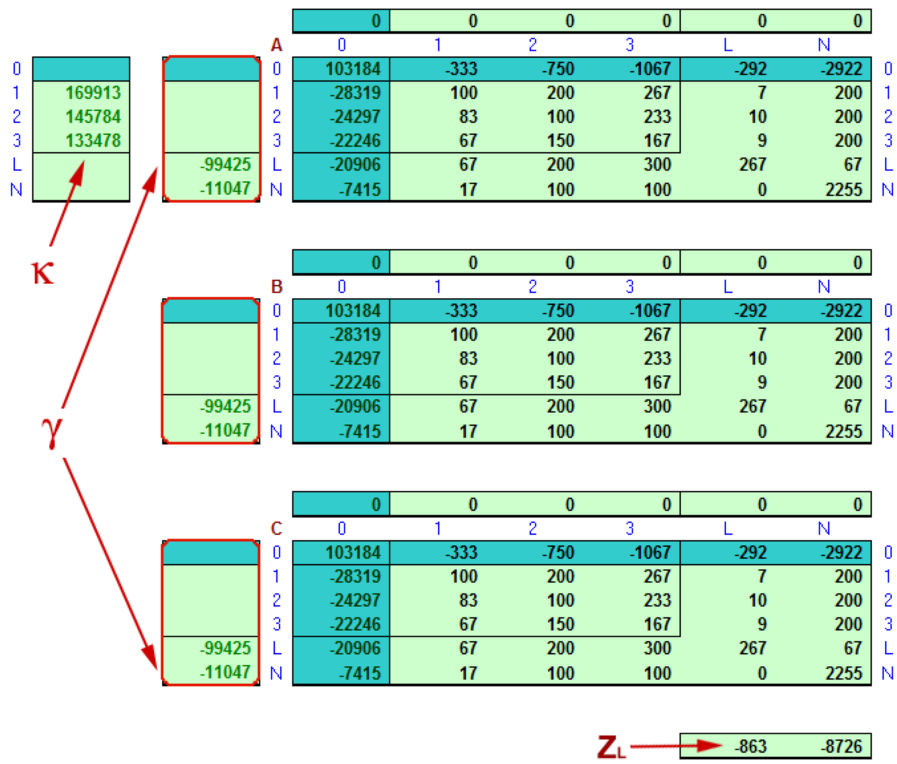


Figure B-2: Model 0's State Variables for One Economy

C: Utility Parameters

Initiation of the model is completed by generating utility parameters for the sectors I. Computation of a sector's hyperbolic description of productive indifference proceeds from observation of an operating decision, ξ, Q_1, Q_2, … , Q_N, where the set [ξ,Q_J] is presumed the optimal instance of the [Y,E_J] set in Equation 3-1's statement of a production function. These are the data specified on the GPE worksheet and automatically brought over to the UTILITY sheet.

Any change to the GPE sheet requires regeneration of the model's underlying structure of utility tradeoffs. The experimenter's only interaction with the UTILITY worksheet is to click the COMPUTE UTILITY button shown in Figure C-1; but this must be done whenever the initial input/output data are changed.

Sectors' Production and Utility Tradeoffs, physical units/year

	E	0 VALUE	1 CAPITAL	2 GENERAL	3 NON_DUR	L BOURGS	N PRLTRT	
VALUE	0	309552	-100	-750	-3200	-876	-8766	0 VALUE
CAPITAL	1		30	200	800	20	600	1 CAPITAL
GENERAL	2		25	100	700	30	600	2 GENERAL
NON_DUR	3		20	150	500	26	600	3 NON_DUR
BOURGS	L		20	200	900	800	200	L BOURGS
PRLTRT	N		5	100	300	0	6766	N PRLTRT

	0 VALUE	1 CAPITAL	2 GENERAL	3 NON_DUR	L BOURGS	N PRLTRT
P	2.0000	1869.04	213.82	45.88	1118.59	20.00

Compute Utility

Figure C-1: Data for Computing Utility Tradeoffs

In restating Equation 3-1 in terms of [ξ,Q_J] we introduce a slight rearrangement in anticipation of the algebraic development for the utility set [Z,U_J]:

C-1)
$$\frac{(Z-\xi)}{Z} = \frac{U_1}{(U_1+Q_1)} \cdot \frac{U_2}{(U_2+Q_2)} \cdot \;\cdots\; \cdot \frac{U_N}{(U_N+Q_N)}$$

The [ξ,Q_J] set is also presumed optimal in regard to an observed price spectrum π, P_1, P_2 , . . . , P_N. Prices [π,P_J] developed on the GPE worksheet are automatically brought over to the UTILITY sheet, and enter the analysis through a reorganization of the Equations 12-1:

C-2)
$$Z = \zeta/\pi + \xi\,; \quad Z - \xi = \zeta/\pi$$
$$U_J + Q_J = \zeta/P_J\,; \quad U_J = \zeta/P_J - Q_J$$

Substituting the right-hand sides of Equations C-2 for their identities in Equation C-1 eliminates all references to the production coefficients:

C-3) $$\frac{\zeta/\pi}{\zeta/\pi+\xi} = \frac{\zeta/P_1 - Q_1}{\zeta/P_1} \cdot \frac{\zeta/P_2 - Q_2}{\zeta/P_2} \cdot \ldots \cdot \frac{\zeta/P_N - Q_N}{\zeta/P_N}$$

Cross-multiplying simplifies Equation C-3 to ...

C-4) $$\frac{\zeta^{N+1}}{\pi \cdot P_1 \cdot P_2 \cdot \ldots \cdot P_N} = (\zeta/\pi+\xi) \cdot (\zeta/P_1 - Q_1) \cdot (\zeta/P_2 - Q_2) \cdot \ldots \cdot (\zeta/P_N - Q_N)$$

Equation C-4 can be further reduced by multiplying through with the left-hand-side's inverse:

C-5) $$1 = (1+\pi\cdot\xi/\zeta) \cdot (1-P_1\cdot Q_1/\zeta) \cdot (1-P_2\cdot Q_2/\zeta) \cdot \ldots \cdot (1-P_N\cdot Q_N/\zeta)$$

Extracting ζ from this equation begins by taking a natural logarithm of each side:

C-6) $$0 = \ln(1+\pi\cdot\xi/\zeta) + \ln(1-P_1\cdot Q_1/\zeta) + \ln(1-P_2\cdot Q_2/\zeta) + \ldots + \ln(1-P_N\cdot Q_N/\zeta)$$

Solving Equation C-6 then requires reference to the series expansion of the natural logarithm. When | a | < 1,

C-7) $$\ln(1+a) = a - a^2/2 + a^3/3 - a^4/4 + \ldots$$

Stating Equation C-6 in terms of this expansion leads to ...

C-8)

$$0 = \frac{(\pi \cdot \xi)}{\zeta} - \frac{(\pi \cdot \xi)^2}{2 \cdot \zeta^2} + \frac{(\pi \cdot \xi)^3}{3 \cdot \zeta^3} - \frac{(\pi \cdot \xi)^4}{4 \cdot \zeta^4} + \cdots$$
$$+ \frac{(-P_1 \cdot Q_1)}{\zeta} - \frac{(-P_1 \cdot Q_1)^2}{2 \cdot \zeta^2} + \frac{(-P_1 \cdot Q_1)^3}{3 \cdot \zeta^3} - \frac{(-P_1 \cdot Q_1)^4}{4 \cdot \zeta^4} + \cdots$$
$$+ \frac{(-P_2 \cdot Q_2)}{\zeta} - \frac{(-P_2 \cdot Q_2)^2}{2 \cdot \zeta^2} + \frac{(-P_2 \cdot Q_2)^3}{3 \cdot \zeta^3} - \frac{(-P_2 \cdot Q_2)^4}{4 \cdot \zeta^4} + \cdots$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \cdots$$
$$+ \frac{(-P_N \cdot Q_N)}{\zeta} - \frac{(-P_N \cdot Q_N)^2}{2 \cdot \zeta^2} + \frac{(-P_N \cdot Q_N)^3}{3 \cdot \zeta^3} - \frac{(-P_N \cdot Q_N)^4}{4 \cdot \zeta^4} + \cdots$$

Taking the first four terms in each expansion of Equation C-8 to approximate the equality, and multiplying through by ζ^4 brings us to Equation C-9, which is a soluble, cubic equation in ζ for which all the coefficients are observed among an economic actor's operating decisions.

C-9)

$$0 = \frac{\zeta^3}{1} \cdot \left[+ (\pi \cdot \xi) + (-P_1 \cdot Q_1) + (-P_2 \cdot Q_2) + \cdots + (-P_N \cdot Q_N) \right]$$
$$+ \frac{\zeta^2}{2} \cdot \left[- (\pi \cdot \xi)^2 - (-P_1 \cdot Q_1)^2 - (-P_2 \cdot Q_2)^2 - \cdots - (-P_N \cdot Q_N)^2 \right]$$
$$+ \frac{\zeta}{3} \cdot \left[+ (\pi \cdot \xi)^3 + (-P_1 \cdot Q_1)^3 + (-P_2 \cdot Q_2)^3 + \cdots + (-P_N \cdot Q_N)^3 \right]$$
$$+ \frac{1}{4} \cdot \left[- (\pi \cdot \xi)^4 - (-P_1 \cdot Q_1)^4 - (-P_2 \cdot Q_2)^4 - \cdots - (-P_N \cdot Q_N)^4 \right]$$

A quadratic approximation is also available on the basis of Equation C-8's first three terms. These two approximations to ζ, together with a knowledge of which is the better of the two estimates, allows formulation of any number of iterative processes by which ζ might be reported to any desired accuracy. Once ζ has been extracted from this system, Z and all the U 's fall out of Equations C-2.

As shown in Figure 9-1, an economy K's utility parameters are reported on its respective worksheet ECON 1, ECON 2, or ECON 3. Whenever the SIMULATE button is clicked, these parameters are read back into the emulation program to provide boundary conditions for the next interval to be simulated. This provides an opportunity for the experimenter to enter a plastic stimulation of his choosing by changing one or more utility parameters.